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ABSTRACT

These eleven essays range across a variety of topics. They concern: (1) philosophy and the curriculum, with comments on accountability, humanism, programmed learning, hierarchies, structure, idealism, existentialism, realism, experimentalism, and essentialism; (2) philosophy in the mathematics curriculum, with comments on mathematics laboratories, programmed learning, contracts, management systems, and learning centers; (3) objectives and learning activities in the mathematics curriculum, focusing on objectives and on using various types of materials; (4) providing for individual differences and designing the mathematics curriculum, with sections on slow learners, talented pupils, learning activities, other ways of providing for individual differences, and scope; (5) principles of learning and the mathematics curriculum, with comments on purpose, interest, problem solving, meaningful learning, individual differences, mathematics concepts, decision-making, and evaluation; (6) mathematics and the learner; (7) change in education; (8) inservice education and the computer; (9) innovation, technology, and the mathematics curriculum; (10) knowledge of results, with comments on evaluation procedures, behaviorism, and humanism; and (11) evaluating achievement in mathematics, including the use of observations, checklists, teacher-made tests, standardized tests, work samples, parent-teacher conferences, tape recorders, pupils' responses, and pupil-teacher conferences. Some chapters provide selected references. (MNS)

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TEACHING MATHEMATICS
IN THE
ELEMENTARY SCHOOL
(A COLLECTION OF ESSAYS)

BY

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PHILOSOPHY AND THE CURRICULUM

Philosophical considerations are certainly inherent in developing the curriculum. It almost appears that each decision made by the teacher is based upon one or more schools of thought in philosophy. In most situations then, a different choice exists, than what was made, in selecting objectives, learning activities, and appraisal procedures.

Accountability of Teachers

The accountability movement is quite prevalent. The lay public, parents included, want to know how well students are achieving in schools. Thus, strong faith is emphasized in testing movements. Standardized tests, statewide tests, district wide tests, and teacher written tests are to provide answers on how well pupils are achieving. Much money and time goes into testing. Observable, demonstrable results are wanted from students as to how much has been learned. Teaching and testing go hand in hand. To be a good teacher means to have measurable results from students. The test results from these students must be high, acccording to standards set by the norms of the standardized tests, as well as by those involved in developing state and district wide tests.

Test results are then compared among the different states. Results of statewide and district wide tests are also compared. Thus, test results within a state provide data for comparing achievement among school districts within the borders of the involved state. Also, test results among the separate schools are compared within a school district. Competition becomes stronger to excel among the states to be the leading state in terms of student achievement. Within a state, a school district wants to be the leader in having the highest test results within that geographical region. Or, within a school district, a separate school wants the published distinction of having the highest test results within the reporting unit.

A career ladder emphasis includes teacher competition to have his/her students achieve well in tests in order to move to the highest level in salary. Salary and prestige objectives are motives in wishing to move higher on the career ladder. If there are four levels in moving higher on the career ladder, students within the classroom of the involved teacher will need to have measurable test results which demonstrate the concept of excellence. Excellence means high test results. The results from the tests are observable and objective. Subjectivity then is not involved in noticing gains in achievement from students.

There is a definite philosophy inherent in the measurement movement. Certainly, there is much faith in that test results truly give us a vital means of ascertaining student progress. A lack of trust exists in utilizing other means to determine learner achievement.

Accountability thus emphasizes the following:

1. It can be determined in measurable terms what students are to learn. There is certainty involved here in believing that specialists can identify that which is essential and vital for student acquisition.

2. The teacher can select those learning opportunities which can aid pupils to attain the stated precise ends. All other stimuli ideally in learning opportunities should be omitted. Only those stimuli in activities and experiences should be emphasized which stress goal attainment by students.

3. There are definite procedures which may be utilized to measure student achievement. What pupils have learned is then measurable and not internal with the learner.

Humanism and the Curriculum

Humanism, as a psychology of learning, emphasizes students choosing, within a framework, which learning activity to pursue and which to omit. There are personal decisions and choices to make by students in terms of which experiences to choose and which to overlook. There, however, must be adequate tasks from which a student may select in order to achieve sequentially and continually.

Humanism emphasizes that learners attempt to achieve self actualization. To attain self actualization, each pupil's physiological (food, shelter, rest, water, safety, love and belonging as well as esteem) needs must be met first. The individual selects means of attaining the optimal self. No other being can do this for the pupil. The involved student makes choices and decisons.

Which methods of teaching then might humanism emphasize as a psychology of learning?

1. Learning centers. An adequate number of stimulating learning centers and tasks must be in evidence. Only then can each student select sequential tasks which meet personal needs. Choices are made on the basis of attaining self actualization or optimal progress.

2. Individualized reading. The pupil, not the teacher, selects which library book to read. A wide variety of topics and titles, as well as levels of complexity, need to be inherent in the library books. A plentiful supply of purposeful library books emphasizes that pupils may select as well as omit reading selected library books which do not meet personal needs and interests. After the completion of reading a chosen book, pupils with teacher guidance may appraise the former's progress in comprehension of content and in attitudes.

3. A contract system. Pupils individually with teacher guidance choose which specific projects to complete. The exact descriptions of each project is listed on the contract. The due date for completion of the project is entered onto the contract. Both the learner and the teacher sign the contract upon agreement as to its terms. Thus, the involved student has considerable input as to the purposes or objectives of the contract. What is to be done to carry out the terms of the contract is planned by the student with teacher guidance. The completed project is appraised by those involved in developing the contract, namely the pupil and the teacher.

Programmed Learning

Programmed learning emphasizes quite a different philosophy of education as compared to humanism. The latter stresses students making sequential choices in terms of learning activities. A psychological curriculum is then in evidence. Toward the other end of the continuum, programmed learning advocates a logical curriculum. Thus, the programmer in developing programmed materials selects subject matter for the student to acquire. The content is divided into small amounts of subject matter to learn in sequential steps. Thus, a student reads a few items, such as several sentences or an arithmetic number pair or fact, responds to a completion item, checks his/her answer with the correct one as provided by the programmer. The programmed items can be presented in textbook or software/ microcomputer form.

A student responding correctly to a completion item is rewarded and is ready for the next sequential item in linear programming. If an incorrect response is given, the learner now knows the correct answer, as provided by the programmer and then is still ready for the next sequential item.

The same/similar procedure may be utilized again and again in programmed instruction: read, respond, and check for each sequential step of learning. With the use of software and the microcomputer, reinforcement of each correct response may well be shown with a smiley face or a statement, such as "that's correct."

Programmed instruction emphasizes:

1. A logical curriculum in that the programmer determines sequential items of content for students to acquire.

2. No input from students in terms of learner-teacher planning of what the former is to learn.

3. Measurement of each step of learning by the student so that incorrect acquired content is not practiced.

Thus, a student knows immediately if he/she was correct or incorrect when giving a specific response.

A Hierarchy of Objectives

Robert Gagné, a leading psychologist, emphasizes that teachers go through sequential steps in planning objectives for pupils to attain. Thus, to develop the sequential precise objectives, the teacher first of all asks himself/herself, "What do I want students to learn?"

Once that broad question has been answered in writing, the instructor writes measurable objectives in sequence which students need to achieve in order to realize the overall goal of "What do I want students to learn?" If the objectives are not in a proper order, the students individually will not be successful achievers. Certain objectives then will not be achieved. Perhaps, one or more objectives need to be written to be ordered within the previously stated ends. Thus, a smaller step of achievement may be in evidence for the learner to move in the direction of attaining the next more difficult objective. Sometimes, the gap is too large between objective A and B and the involved student is foreordained to experience failure. Placing one or more objectives between A and B can make for successful student learning. Objectives then need to be arranged in ascending order of difficulty for optimal progress to be in evidence, for the involved student.

Robert Gagné emphasizes:

1. measurable sequential objectives for students to achieve.
2. teachers writing the ordered objectives in a logical curriculum.
3. pupils being successful in achieving each objective due to appropriate sequence in learning.

Jerome Bruner and the Structure of Knowledge

Jerome Bruner, psychologist from Harvard University, advocates students achieving structural content in each curriculum area. The structure of knowledge should be identified by academicians in their academic area of specialty. Thus for the social studies curriculum, social scientists such as historians, geographers, political scientists, anthropologists, sociologists, and economists on the college and university levels would select the structure of knowledge (generalizations) from each of their respective areas of specialty. Structural ideas provide the framework for any subject matter area. The content indicates how the generalizations (broad ideas) are related. Too frequently, students have learned isolated subject matter. Instead, Jerome Bruner advocates that structural ideas identified by academicians represent related subject matter as perceived by the involved academician, such as the historian as an example. Trivia would then not be learned by students since vital content has been carefully selected by the involved academician. The structural ideas then would be available to teachers. Teachers may guide students to achieve these structural ideas in the latters own words. A spiral curriculum would be in evidence since students may attain the structural ideas at each successive level of complexity as they progress through the different levels of schooling.

The social studies teacher also needs to stress methods of acquiring structural ideas as advocated and utilized by academicians. Thus, for example, primary and secondary sources as used by the historian should also be utilized by students on the elementary and secondary levels of schooling. With emphasis being placed upon methods of study and structural ideas promoted by academicians, students may achieve key ideas inductively.

Jerome Bruner recommended that structural ideas be achieved through induction by using enactive (concrete), iconic (illustrations using diverse audio-visual aids), as well as symbolic (abstract) learnings in achieving sequential ideas.

Bruner deemphasizes:

1. deductive (lecture, expositive, and heavy use of explanations in teaching students) methods of instruction.

2. abstract (reading and writing) learnings comprising the curriculum. Rather, enactive, iconic, and symbolic procedures need to be utilized in sequence.

3. isolated, trivia content being stressed in any given curriculum area.

Idealism in the Curriculum

The late Herman Horne (1874-1946) was a leading advocate of idealism as a philosophy of education. Dr. Horne emphasized that mind is real. The mind or soul survives the body after death. More than one lifetime for individuals is necessary in order to move closer to the Absolute or the universal mind.

An idea centered curriculum is necessary in moving from the finite to the Infinite.

A liberal arts general education is advocated by Dr. Horne. Thus, history and geography in the social studies, the fine arts, religion, philosophy, the language arts, and the sciences provide content for general education for all students. Thus, the mind is liberated in a liberal arts curriculum. Vocational education is definitely not recommended. This can come later after a quality general education for students has been secured.

In general education, universal ideas are developed, rather than specific, isolated facts. Mental development is highly important for learners. Only then does the mind become significant in the curriculum. The limited person must achieve more in the direction of the Absolute.

Dr Horne would not advocate education stressing:

1. vocational education for students. Rather, a liberal arts general education curriculum needs to be in the offing.

2. object centered teaching-learning situations such as in science units, and thus minimize literature, religion, history, geography, the fine arts, and philosophy, in particular.

3. the physical facets of human development as being superior to mental achievement in the curriculum.

Existentialism in the Curriculum

Existentialism emphasizes pupils making choices and decisions in the curriculum. Restraints on learners must truly be minimal to stress an existentialist philosophy of learning. The individual first exists and then chooses goals to make the self (essences). Each person did not ask to be born. Rather, each human arrives on the scene and then is forced to make choices. Goals to attain are really not given to any person. Rather, they must be chosen and selected freely if the desire is there or not.

Each choice and decision is made in a ridiculous environment. Life in and of itself is not rational. It is absurd instead. But within the absurd, life continues and values must be chosen. Uneasiness and anxiety are inherent in persons when choosing occurs. It is overwhelming to select among alternatives in daily life. But to choose means to be human. The individual can permit others to make decisions involving oneself. However, that person ceases to be a human being. To make decisions is to be human.

Decisions made are subjective and definitely not objective. Curriculum areas that contain subjective content are superior to those that possess so called objective subject matter. Thus, the fine arts, the humanities, history philosophy, and religion indeed become significant curriculum areas.

The individual chooses and makes choices. He/she cannot blame others for what occurs and transpires. The decision maker then is responsible for choices made. No other person need be blamed if the results of a decision turn out to be negative.

An existentialist curriculum does not emphasize:

1. measurable objectives for students to attain.
2. learning activities selected by the teacher.
3. evaluation procedures chosen by the teacher.

Rather, as much as possible, students individually need to select goals, activities and experiences, as well as appraisal procedures. The student is responsible for his/her own personal life and thus must make responsible decisions.

Realism in the Curriculum

A realist believes that a person can know all or part of the real environment as it truly is. The learner does not receive ideas only about the natural/social environment but can know reality as it is. He/she may actually receive a replica or blue print of what has been observed, heard, tasted, touched, and felt.

Since the individual can know the real environment as it truly is, the curriculum area of science becomes relevant indeed. General science or the separate academic disciplines of astronomy, biology, chemistry, geology, zoology, botany, and ecology may well provide content which provides learnings as the world truly is.

Mathematics which is precise and has patterns can also provide content which can be known as it truly is. Logical thinking provides accurate content to students. Mathematics is also the language of science. With precise statements made pertaining to phenomena in science, accurate content in mathematics may describe completely and comprehensively objects in the scientific realm.

Realism emphasizes:

1. individuals being able to know phenomena the way it truly is. A replica or duplicate of the environment then exists in the senses of the human being.

2. the relevance and significance of science and mathematics in the school curriculum. The involved subject matter here is objective and quantifiable.

3. accurate content, not subjective ideas. Subject matter learned must be objective independent of the observer.

4. subject matter from history and geography also contains objective content. History with its names, dates, places, and events centers itself around content knowable by the knower as it truly is. The same is true of geography, with its geographical features of mountains, rivers, lakes, oceans, plains, plateaus, peninsulas, and seas.

5. even the subject area of values and morals can be objectified. Thus, a set of values/morals can be agreed upon if they have stood the test of time. Subjectivity in values and morals where truth is in the eye of the beholder needs to be avoided.

Experimentalism and the Curriculum

Experimentalists emphasize the world of experience. Any person can only know what he/she has experienced. The real world as it truly is can not be verified through experience. Nor does one only know ideas about the real world as idealists emphasize. Rather, what is significant is experiences of human beings.

Each person experiences problems. The problems need indentification. Information or data needs to be acquired directly related to the problem being solved. Content is not learned for the sake of doing so, that is knowledge for its own sake, but, subject matter is instrumental. Instrumentality of subject matter is involved when it is utilized to solve problems. Once adequate data has been acquired, then an hypotheses or possible answer to the problem should be attained. Hypotheses are tentative, not absolutes, and subject to testing in in real life situation. The hypotheses may need to be revised as the consequences of results indicate it is necessary to do so. The complete act of thought is involved within the framework of problem solving.

Life in school should not be separated from that which exists in society. school and society are integrated, not isolated entities. A utilitarian curriculum is then in evidence. Society has its problems. The problems need identification and solutions.

In society, groups identify and solve problems. Thus, in the school curriculum, committees need to engage in problem solving experiences.

Too frequently, schools have separated the curriculum from the learner. Physical punishment or methods of embarrassment have been utilized to encourage student learning. The student then becomes separated physically, intellectually, socially, and emotionally from the curriculum. Rather, the teacher needs to provide interesting activities which promote effort in learning. Interest and effort then become one and not separate concepts. The interests that pupils possess provide the effort or encouragement for learning.

Experimentalists do not emphasize:

1. a school curriculum separated from those activities and experiences useful in society.

2. dualisms such as interest and effort, school and society, and the student versus the curriculum.

3. subject matter learned outside the framework of problem solving situations. Subject matter then becomes a means to an end and that end being to solve vital problems.

4. drill and practice per se in teaching and learning. Rather, subject matter must be meaningful and instrumental to that which is perplexing and needs solutions.

Essentialism and the Curriculum

The late William Chandler Bagley (1874-1946) emphasized essentialism in the curriculum. Common learnings vital to and for all students emphasize the heart of essentialist educators. Essentialism may be thought of as general education. Those facts, concepts, and generalizations needed by students in general comprise the essentials.

Dr. Bagley advocated the teaching of selected academic areas which had stood the test of time. The changeless, rather than the changing curriculum was emphasized. The following stable academic areas were to be taught:

1. history and geography. These two academic disciplines have been taught through the centuries to students. Other social science disciplines, such as sociology and anthropology, have subject matter which was recognized more recently as being significant. However, there is much more agreement as to salient objective content that should be taught in history and geography. These two academic disciplines also have precise, exact content to be taught. History, for example, has specific significant items for pupils to learn, such as events, places, and names.

2. grammar, writing, and oral communication. The curriculum area of grammar with its parts of speech and how words are utilized in sentences provide essential learnings for students to become increasingly proficient in writing and speaking. Through teacher determined objectives, pupils can become proficient in communication skills.

3. science and mathematics. The precise disciplines that provide subject matter for both science and mathematics need emphasis. The exact disciplines, not opinions, need to be adequately emphasized in teaching and learning. Vital subject matter must be acquired by students.

4. reading and spelling. These two curriculum areas are basic to learning in the previously named academic areas in items one, two, and three above. For example, pupils read to learn in each academic discipline.

5. firm discipline. Obedience on the part of students is important. With excellent discipline and obedient pupils, teachers may then teach and students learn.

Dr. Bagley opposed:

1. an activity centered curriculum which takes excessively much time to implement. A well disciplined classroom provides students with ample opportunities to learn the basics.

2. pupil-teacher planning. The teacher is educated and trained to teach. Students lack that maturity. Thus, the teacher needs to select essential objectives, learning activities, and appraisal procedures.

3. career education curricula. Rather, a subject matter curriculum needs implementation. There are basic learnings for each student to acquire. After the basics have been mastered by all pupils, then careers can be explored and emphasized. But first, essential subject matter needs to be attained by each pupil.

In Summary

There are diverse philosophies which may be stressed in the curriculum.

1. Accountability movements advocate that measurable results are required from students. Teacher proficiency is demonstrated in terms of measured demonstrable achievement from each student.

2. Humanism stresses students make choices to terms of which objectives and learning activities to pursue.

3. Programmed learning emphasizes students progress in small sequential steps as determined by the programmer. Feedback to students in terms of each correct response given is provided continuously and immediately.

4. Robert Gagne's hierarchy of objectives stresses pupils achieving measurable objectives carefully sequenced by the teacher.

5. Jerome Bruner believes that students need to achieve structural ideas as identified by professional academicians. The structure of knowledge should be learned inductively through enactive (concrete), iconic (semi-concrete), and symbolic (abstract) materials.

6. Idealists stress the importance of general education for all students in which the latter move from the finite to the infinite being. Vocational education is not recommended until a comprehensive program of general education has been completed.

7. Existentialists believe that students need to make choices and decisions. Life itself is absurd and ridiculous. Choices and decisions by students within a flexible curriculum need to be in the offing.

8. Realists believe that students in part can know the real world as it truly is. Thus, reality in the worlds of science and mathematics, in particular, need depth teaching. Subjective content in the curriculum needs to be avoided.

9. Experimentalists advocate students acquire subject matter to solve problems. Facts, concepts, and generalizations learned are instrumental. The concept instrumental emphasizes the utilization of subject matter to solve problems. Subject matter then is not learned for its own sake.

10. Essentialists believe there is a common core of subject matter which is stable and necessary for all students. This is basic subject matter and is definitely not related to those academic areas which are elective courses.

PHILOSOPHY IN THE MATHEMATICS CURRICULUM

Too frequently, teachers follow methods of teaching involving the rather heavy use of mathematics textbooks, workbooks, and worksheets. The involved teacher may stress a lecture/explanation approach in teaching - learning situations.

Additional philosophies need to be studied and analyzed. Hopefully, teachers and supervisors will develope a philosophy which truly guides students to achieve optimally in the mathematics curriculum.

A Mathematics Laboratory

Students may learn relevant facts, concepts, and generalizations when using a learning by doing approach. Ample concrete materials should then be in evidence in ongoing lessons and units. Items for weighing, measuring, counting, and determining volume can be used by students in real life situations. An active, not passive, learner is choosing objectives and learning activities in a flexible mathematics curriculum. The teacher does not determine sequential learnings for students. Sequence resides within the student in ordering what is to be learned. The teacher, however, is not left out of the picture. He/she selects materials and suggests possible tasks for students to pursue. There are numerous manipulative materials available for students to utilize and learn vital subject matter.

Ediger[1] wrote:

> Pupils should have ample opportunities to experience the mathematics laboratory concept of working. The mathematics laboratory emphasizes tenets of teaching and learning such as the following:

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[1] Marlow Ediger, The Elementary Curriculum, a Handbook, Kirksville, Missouri, Simpson Publishing Company, 1977, page 170.

(a) Pupils are actively involved in ongoing learning activities.

(b) A variety of experiences is in evidence so that pupils may select materials and aids necessary for problem solving.

(c) Practical experiences are emphasized for learners in that they actually measure the length, width, and/or height of selected people and things; weigh real objects and record their findings; find the volume of important containers; as well as determine areas of selected geometric figures.

(d) Pupils become interested in mathematics due to reality being involved in ongoing learning activities.

(e) Provision is made for individual differences since there is a variety of learning opportunities for pupils from which to select on an individual basis.

(f) Meaning is attached to what is being learned since pupils individually and in committees work on tasks adjusted to their present achievement levels.

A mathematics laboratory philosophy of learning deemphasizes the following:

1. teachers solely selecting objectives for students to attain.

2. a logical curriculum in which instructors sequence learning activities

3. student responding to stimuli rather than choosing goals and experiences.

4. heavy use of abstract materials as compared to utilitarian outcomes in the mathematics curriculum.

5. teachers evaluating student progress as contrasted with learner self appraisal of achievement.

Programmed Learning in Mathematics

Programmers determine what students are to learn in an ordered sequence. This is true if programmed books or computerized instruction is in evidence. A small bit of subject matter is learned prior to the involved student responding to a test item involving a brief response. The student is then ready to check his/her response with that provided by the programmer. A correct response is rewarded and reinforcement is then in evidence. The student responding incorrectly sees the correct response as provided by the programmer and is also ready for the next linear sequential programmed item. A same/similar method is utilized again and again in programmed instruction -- read, respond, and check. With computer assisted instruction (CAI), a smiley face may appear on the screen if a learner has responded correctly to an item. If an incorrect response is is given in CAI, the involved learner will be asked to respond again, as indicated in print on the screen. With a second incorrect response, the student will generally then be provided the correct answer and he/she may then progress to the next linear task. Many programs of software to use with computers will indicate at the end of a lesson or unit, the percent of initial correct responses given by learners, as indicated on the screen. If a second attempt was made to offer a correct response to CAI, this also will be tabulated and shown on the screen. Thus, a teacher may monitor each student's performance in using CAI by noting what percent of the items each student gave correct responses to in a given lesson or unit.

Woolfolk and Nicholich[2] wrote the following:

> . . . The terminal might be as simple as a teletypewriter (like an electric typewriter) or as complicated as a cathode ray tube display terminal (like a television screen mounted above an electric typewriter). The computer presents problems or questions. The student then answers, by typing the response or by writing on the display screen with a special light pen. The computer responds to the answer, either by typing a message back to the student or by presenting a message on the display screen. If the student gives a wrong answer, the computer might do one of several things, depending upon the complexity of the program. The computer could (1) say, "Wrong, try again"; (2) point out the specific error; (3) branch into a remedial lesson; (4) ask the student to call the teacher for help; (5) give the right answer and present another problem.
>
> Besides presenting lessons, computers can automatically keep records of which units each student has completed, how long it took each student to complete each lesson, which questions were answered correctly, and which questions presented the greatest problems. All lessons done on the computer are automatically graded for the teacher. (If in the future students have computer terminals in their homes, homework could be done on the terminal, graded automatically, and never be "left at home"!) These record-keeping and grading uses of the computer can save teachers quite a bit of time.

Each student may achieve at his/her own optimal rate of achievement in utilizing programmed materials. It is a completely individualized program of instruction in mathematics.

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[2] Anita E. Woolfolk and Lorraine McCure Nicolich, Educational Psychology. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1980, page 154.

Programmed instruction deemphasizes:

1. student-teacher planning of content for the former to acquire.

2. learners determining the order of subject matter to be attained.

3. inexact content to be learned. An answer to a programmed item is either correct or incorrect. Personal feelings and values are omitted.

4. learners being involved in developing criteria for self-appraisal purposes.

5. student-teacher planning of learning activities for the former to pursue.

A Contract System

A completely individualized plan of teaching mathematics, but having a quite different philosophy than programmed learning is a contract system. The involved student with teacher assistance may plan what he/she desires to learn and complete. The tasks cooperatively determined by [illegible] student and teacher are written in an agreement referred to as a contract. Clarity in writing the activities is important. The interests and purposes of the student are reflected within the stated learning activities.

The due date, as well as the student and teacher's signatures are affixed to the contract.

Pertaining to a contract system, Jarolimek[3] wrote:

> A pupil contract is defined as a cooperatively developed agreement between an individual pupil and the teacher. The contract specified precisely what the pupil will do and when the work is to be completed. Because contracts are individually negotiated, the nature and extent of the work to be completed can be well suited to the ability level and the interests of the pupil. Some teachers develop and use a contract form that gives the agreement an official appearance. Contracts are signed by both the pupil and the teacher.
>
> The use of contracts has educational values for pupils beyond being an interesting assignment format. The pupils should learn what is involved in a contractual agreement and that there usually are adverse consequences when contracts are not completed as specified.

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[3] John Jarolimek, Social Studies in Elementary Education, Sixth edition. New York: The Macmillan Company, 1982, page 123.

Opposite of a contract system are the following statements:

1. the teacher selecting objectives, learning activities, and evaluation procedures for students.

2. programmed learning with its predetermined content for students to acquire.

3. a highly structured curriculum in which specific textbooks, workbooks, and worksheets are used in teaching.

4. large group instruction used to teach students.

5. an adult determined curriculum.

Management Systems of Instruction

Behaviorist advocate using management systems of teaching students. Mastery learning, instructional management systems (IMS), and individually prescribed instruction (IPI) reflect management philosophies in the mathematics curriculum. Each emphasize the utilization of precise objectives in instruction. Measurable objectives are then arranged in ascending order of complexity. The teacher selects learning activities so that students individually may attain the chosen objectives. After a precise outcome has been achieved, the involved learner is ready to attain the next sequential end. It is certain if a student has/has not attained a specific objective. Guesswork is not inherent.

Pertaining to the utilization of precise ends in teaching, Kibler, et. al.[4] wrote:

> 1. Several studies have demonstrated that students can be more efficient learners if they are provided with objectives (Mager and McCann, 1961; McNeil, 1966; Miles, Kibler, and Pettigrew,1967). Perhaps one of the reasons students do better when given copies of the objectives of a unit of instruction is that they are spared the frustration and time-consuming effort of trying to guess what the teacher expects of them.

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[4] Robert J. Kibler, et. al. Behavioral Objectives and Instruction. Boston: Allyn and Bacon, Inc., 1970, pages 22 and 23.

2. Given such clearly specific objectives, curriculum planners are better able to arrange sequences of courses or units of instruction. Knowing what students (hopefully all students) are able to do at the end of courses and what students are able to do at the beginning of courses (prerequisites), it should be possible to eliminate unnecessary overlap of courses and to identify and fill in gaps between courses.

3. Students and their advisors are able to plan their course programs better when they can read course descriptions which include informational objectives.

4. Through clear behavioral objectives, teachers are able to tell other teachers what they teach. Stating that "students learn to name each state and its capital in the United States" tells considerable more than stating "United States geography is taught."

Opposite of measureable objectives in teaching would be the following:

1. general goals which leave leaway in interpretation as to their achievement by students.

2. student-teacher planning of objectives.

3. choices made by students in terms of media to be used in learning.

4. students ordering or sequencing their own personal objectives to attain.

5. learners being involved in selecting means of evaluating their own achievement.

Learning Centers

Humanism, as a psychology of learning, advocates learner imput in selecting objectives, learning activities, and appraisal procedures. A learning centers approach is one method of emphasizing humanism in the curriculum. The following are given as suggested titles of learning centers: problem solving; construction of models; textbooks and workbooks; programmed learning; microcomputer use; geometry; algebra; and creative endeavors.

A task card may be placed at each center with an ample number of tasks contained therein. The involved learner can then choose sequential tasks to complete. Enough activities need to be available so that a student might omit undesired experiences.

Pertaining to humanism in the curriculum, Zais[5] wrote:

> A concept that defies precise definition, education is probably most succinctly described as the process of actualizing human potentials. As previously stated, it poses the question: What can man become? A notion of what education is may be best conveyed in metaphoric terms. Thus, the educated man often is viewed as a person who has transcended the psychological contraints of the culture, yet who retains membership in the human community. At the same time he is able to grapple with the open endedness of existence in the search for meaning.
>
> Some describe education as the process of expanding awareness and extending one's existence to an ever greater sense of Being. Others propose that education is the process of defining and redefining oneself in terms of successive reassessments of one's transactions with others and the environment. Still others maintain that education concerns itself with the "big" questions of human existence: What is real? What is man? What is good? Implicit in all these statements is the notion of "liberation" (embodied in the term "liberal education") and the position that the specific outcomes of education are always in question.
>
> A curriculum based on the humanistic model then, would be structured around "openness." Outcomes would tend to be stated in humanistic rather than behavioral terms, and the behavioral psychology of the training paradigm would be supplanted by the "self" or "existential" psychology of Abraham Maslow or Carl Rogers. Rather than specifying the psychological principles which would govern teachers' manipulation of learners, the education curriculum would call for "authentic" interpersonal relationships among learners and between teacher and learners. The "motivation" and behavior change of learners would be intrinsically stimulated, and teachers would function in the role of "sensitive" and "responsive" whole human beings.

A learning centers philosophy emphasizes that each student may choose and engage in the making of decisions. Self-actualization is a significant goal in that a learner might select to complete what is desirable and purposeful. A humane mathematics curriculum is then in evidence.

Opposite of the use of learning centers is

1. predetermined objectives and learning activities selected by the teacher for student attainment.
2. heavy emphasis upon textbooks and workbooks in teaching situations.
3. teacher solely appraising learner progress with no student involvement.
4. a formal, structured mathematics curriculum for students to encounter.
5. a strict learning environment governing student behavior.

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[5] Robert S. Zais, Curriculum, New York: Thomas Y. Crowell Company, 1976, pages 317-318.

In Summary

Teachers and supervisors of mathematics need to study diverse schools of thought involving philosophies of education. Each approach needs to be evaluated in terms of contributing strengths and weaknesses in teaching mathematics. One or more desired philosophies might then be incorporated in ongoing lessons and units.

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OBJECTIVES AND LEARNING ACTIVITIES in THE MATHEMATICS CURRICULUM

Part One: Objectives in the Mathematics Curriculum

Teachers, supervisors, and principals need to select relevant objectives for pupils to achieve. A recommended philosophy in the teaching of mathematics is needed prior to the time that these important goals are to be chosen. With much study and thought pertaining to teaching mathematics, educators may develop a philosophy of teaching containing the following strands of thought:

1. Each pupil in the school setting must achieve to his/her optimum in the elementary school mathematics curriculum.
2. Diverse learning opportunities should be provided to allow for different learning styles which pupils individually possess.
3. Critical thinking, creative thinking, and problem solving need to be stressed in the mathematics curriculum.
4. Pupils should be guided to develop well socially in all curriculum areas in the elementary school.
5. Ample opportunities must be given to aid pupils in learning using both inductive and deductive approaches.
6. Experiences obtained by pupils should be interesting, and meaningful, as well as purposeful.
7. Each child should have feelings of success in the mathematics curriculum.
8. Pupils with teacher guidance should diagnose learner difficulties in the mathematics curriculum and thus work toward remedying these identified deficiencies.
9. There should be rational balance among understandings, skills, and attitudinal objectives pupils are to achieve.

10. The total development of the child should be emphasized such as physical, social, academic, and emotional in ongoing units of study in mathematics.
11. The mathematics curriculum must frequently be appraised by teacher, principals, and supervisors to keep the curriculum area updated. Thus, faculty meetings, workships, and departmental meetings become important in modifying and revising the elementary school mathematics curriculum.
12. Learners should experience realistic situations in ongoing units of study.

Objectives and Mathematics

After careful study and thought, the elementary teacher may identify and implement general understandings objectives such as the following in elementary school mathematics for pupils to attain:

1. the commutative property of multiplication and addition.
2. the associative property of multiplication and addition.
3. the distributive property of multiplication over addition.
4. the identity elements for addition and multiplication.
5. the property of closure for addition and multiplication.
6. the inverse operations of subtraction and division.
7. relationships among operations such as multiplication pertains to repeated addition, and division refers to repeated subtraction of equal amounts.
8. the problem solving method.
9. reference sources available to solve problems in elementary school mathematics.
10. methods used to have pupils evaluate their own achievement.

It would not be adequate to have pupils achieve relevant understandings objectives in the mathematics curriculum only. Pupils also need ample opportunities to acquire important skills objectives. Thus, learners may attain skills such as the following as they progress through the elementary school years:

1. perform addition, subtraction, multiplication, and division operations meaningfully and accurately.
2. identify and diagnose difficulties in performing these basic operations and attempt to remedy deficiencies.
3. draw accurately squares, circles, rectangles, triangles, and other important geometrical figures.
4. draw and measure angles accurately in geometry.
5. use the number line when applicable to get needed content to solve problems.
6. become proficient in thinking critically and creatively in the mathematics curriculum.
7. use appropriate algorithms which harmonize with one's own style of learning in solving problems in the area of mathematics.
8. use a variety of learning aids and activities to aid in achieving optimal development in the area of mathematics.
9. be able to work well with others on committees and at diverse learning centers.
10. analyze and obtain needed content to solve work probelms.

Another kind of objective that is important to emphasize in teaching-learning situations is attitudinal, or affective objectives. If pupils achieve attitudinal, or affective objectives, they will do better in acquiring needed understandings and skills. Teachers, principals, and supervisors need to carefully select attitudinal goals which learners are to achieve. Relevant objectives pertaining to attitudes which learners are to achieve may include:

(a) respecting thinking of other learners.
(b) wanting to compute mathematics content accurately.
(c) desiring to participate effectively in committee work.
(d) appreciating orderliness of the Hindu-Arabic system of numeration.
(e) appreciating, among others, the contributions of the Egyptian, Roman, and

Mayan systems of numeration.

(f) desiring to work to one's own optimal level of achievement in mathematics.

(g) appreciating the contributions of mathematicians in the arithmetic, algebra, and geometry curriculum.

(h) wanting to think creatively and critically in problem solving activities.

(i) developing an appreciation for working in bases other than base ten in arithmetic, such as performing operations in base two, base five, and base eight.

(j) contributing fully in the selection of objectives, learning experiences, and evaluation procedures when teacher-pupil planning is utilized in the class setting.

Specific Objectives in the Mathematics Curriculum

Selected teachers, principals, and supervisors emphasize that objectives for pupils to achieve should be stated in a specific manner. Thus, specific objectives in mathematics may follow the following criteria:

1. It can be measured if pupils have or have not attained the desired objectives.
2. Learning activities may be selected which will guide learners in achieving these precisely stated objectives,
3. Pupils and the teacher can observe if the former have achieved the desired criteria or objectives.

The following are examples, of specific objectives in the mathematics curriculum:

1. The pupil will add correctly the basic number pairs of 4 + 4 = __; 3 + 6 = __; 2 + 5 = __; 6 + 3 = __; and 5 + 2 = __.
2. The pupil will utilize two algorithms in solving each of these problems:

24	32	52	34	41
x42	x23	x22	x22	x22

3. The pupil will solve correctly 90 per cent of the addition problems on page 87 of his/her textbook.
4. The pupil will solve correctly four out of five work problems on page 90 of the textbook.
5. The pupil will write five word problems and include in each an irrelevant item not needed in working toward desired solutions.

In analyzing each of the previously stated objectives, the first specific objective involves recall of content in giving sums to each basic number pair in addition. The second objective deals with a higher level of cognition other than recall of content. Pupils need to use more than one method to solve each problem. The third objective is not clear in and of itself as to what pupils are to achieve. The reader of this objective would need to look at the mathematics textbook being utilized in the class setting to determine the level of achievement being stressed in teaching-learning situations. The fourth objective could involve complex problem-solving activities when encountering the solving of word problems. The fifth objective could stress creative writing on the part of each pupil in the writing of word problems. Irrelevant content would also need to be written in each word problem. Thus, the reader may notice that objectives in mathematics may be written to emphasize

(a) recall of content.

(b) comprehension of information to solve problems.

(c) problem solving.

(d) creative thinking as well as critical thinking.

It behooves the elementary school mathematics teacher to have pupils achieve at a higher level of thinking than the recall level. Thus, critical thinking, creative thinking, and problem solving must be stressed in teaching-learning situations in the mathematics curriculum.

In Summary

Teachers, principals, and supervisors need to develop a philosophy of teaching mathematics which make ample provision for each individual pupil to realize his/her optimal achievement.

A relevant philosophy of teaching pertaining to the teaching of elementary school mathematics will stress the importance of pupils achieving important understandings, skills, and attitudinal objectives. Each objective that pupils are to achieve should be weighed against alternative goals in making the final selection so that learners truly learn what is important in the school and class setting.

Selected teachers, supervisors, and principals may wish to emphasize the use of general objectives in teaching-learning situations. Other educators may rather desire to use specific, measurable objectives. What is of utmost importance in teaching and learning is that pupils attain quality objectives which emphasize higher levels of cognition such as critical thinking, creative thinking, and problem solving. Thus, the dichotomy between general versus specific objectives may be resolved by selecting those objectives which will guide a learner in becoming a fully functioning member in society.

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Part Two: Learning Activities in the Mathematics Curriculum

Mathematics teachers in the elementary school first of all need to select relevant objectives for pupils to achieve. These objectives should emphasize rational balance among understandings, skills, and attitudinal objectives. After these aims have been chosen, the teacher must guide pupils in attaining these objectives through learning opportunities that are

(a) challenging and interesting.

(b) rewarding and satisfying.

(c) purposeful and meaningful.

(d) individualized and provide for each learner.

(e) sequential from the learner's point of view.

(f) inquiry oriented, and yet deductive learning is also emphasized.

(g) characterized by child growth and development tenets.

(h) geared to continuous progress on the part of each learner.

Specific Learning Activities to Attain Objectives

The teacher of elementary school mathematics must select learning opportunities which guide each pupil to achieve optimal development in arithmetic, geometry, and algebra. Possible learning experiences for pupils will now be discussed.

Utilizing the Textbook

Reputable mathematics textbooks can and do provide valuable experiences for pupils in the school and class setting. Textbooks ultimately selected for adoption in teaching mathematics should follow criteria such as the following:

1. Proper order of learning for pupils is in evidence.
2. Adequate illustrations and diagrams are inherent to help pupils understand mathematical concepts, facts, and generalizations.
3. The textbook captures pupil interest and appeal.
4. Key structural ideas are emphasized such as the commutative property of addition and multiplication, the associative property of addition and

multiplication, identity elements for addition and multiplication, the property of closure, and the distributive property of multiplication over addition.

5. The teacher's manual section presents numerous suggestions for teaching-learning situations such as objectives or goals, learning activities, and appraisal procedures.
6. The authors are reputable from the point of identifying relevant learnings for pupils to achieve in mathematics.
7. Adequate attention is given to guide pupils to develop proficiency in problem solving and using various algorithms in computation.
8. Opportunities are given for pupils to utilize what has been learned previously.

Too frequently, all pupils are on the same page at the same time when textbooks are utilized in teaching-learning situations. The teacher needs to individualize instruction. Thus, pupils are assessed to determine their individual present level of achievement within the confines of the adopted textbooks. Each pupil may then be working at a different achievement level as compared to other learners in the class setting. For example, in a fifth grade class, a few learners may need to use third or fourth grade textbooks in order to work at their present level of achievement and thus work in the direction of experiencing continuous progress. As further examples, pupils A, B, and C as revealed by the preassessment may begin on pages 15, 80, and 100 respectively in the fifth grade mathematics textbook. A few selected learners may need to utilize sixth and seventh grade texts in teaching-learning situations since this is their present achievement level. Once each child's present achievement level in mathematics has been determined at the beginning of a school year, the teacher then needs to guide pupils to experience continuous progress. The teacher, during the time devoted to mathematics instruction, would teach, stimulate, and supervise each child to achieve optimal growth and development.

Utilizing the Flannel Board

Primary grade pupils, in particular, may develop many learnings in elementary school mathematics through the use of a flannel board as a learning activity. The teacher on the first grade level, for example, may place two triangles in one set and three triangles in another set. These cutouts may be made of flannel or felt. Pupils may be asked how many members there are in the first set. After responding correctly to this question, learners may be asked to give the number being represented in the second set. The two sets may then be combined with pupils stating the number of members making up this new set. The order of presenting the two previously named sets may then be changed. Pupils then realize $2 + 3 = 3 + 2$. Children are realizing in these learning experiences that the order of addends considered in adding does not affect the sum (commutative property of addition).

If pupils are not ready for addition of two one-digit addends, the following learnings must come in prior sequence:

1. rote counting. Here pupils say the counting numbers in their proper order such as "one," "two," "three," "four," "five," and so on.
2. rational counting. A child points to a first object like a crayon and says "one," points to second crayon and says "two," the third crayon and says "three," and so on. Rational counting is more complex for learners as compared to being able to count in a rote manner. Objectives must be arranged in proper sequence so they can be achieved by pupils.

Cutouts made of flannel or felt for the flannel board should vary in terms of the following:

1. color of cutouts. Pupils desire variety in learning experiences; thus, appealing red, blue, yellow, orange, and green cutouts should be utilized.
2. geometric form of cutouts. Triangles, circles, squares, rectangles, and

parallelograms should be available for use in teaching-learning situations.

Cutouts of diverse pictures may also be used with the flannel board. These pictures, for example, may also be used in helping pupils understand the operation of addition (two jet planes in one set and three jet planes in the second set make a total of how many?) Thus, pictures of animals, cars, trucks, buses, boats, and people may be used in ongoing units of study in mathematics. Each picture should have a small piece of sandpaper pasted on the back so that it will stay attached when placed on the flannel board.

Utilizing Markers

An ample supply of markers should be available to help pupils achieve optimal development in mathematics. Markers such as the following may then be utilized in providing interesting, meaningful, and purposeful experiences:

1. bean, pea, and corn seeds.
2. beads and buttons.
3. crayons, chalk, and pencils.
4. pop caps and tongue-depressor sticks.

These markers may be utilized in the following ways:

1. have pupils count the number of markers in a set.
2. have pupils write numbers that relate to the number of members in each set in addition. The operations of subtraction, multiplication, and division, of course, may also be taught using selected markers.
3. have pupils place the number of markers in a set as specified by the teacher in a given learning experience.
4. guide pupils in understanding the process of regrouping and renaming with the use of these markers. If pupils, for example, are to understand the meaning of 22 - 13 = __, then appropriate experiences should be provided. Since three markers cannot be taken from two markers in the units column, a set of ten markers may be regrouped from the two tens column

and joined with the two ones. The result is a set of twelve markers in the ones column. Pupils may now take three markers from the twelve markers in the units column. Nine markers are then left in the ones column. If a set of ten is taken from a set of ten markers in the tens column, none is left. Thus, 22 - 13 = 9.

Using Place Value Charts

Place value charts may be made very inexpensively in the class setting. Construction paper may be utilized here. Separate pockets may be made to hold congruent slips of construction paper. These pockets must hold the slips of paper for the ones, tens, and hundreds pockets. Thus, if paper strips for the value of twenty-three are to be represented in the place value chart, the learner may place three pieces in the ones pocket. Two sets each containing ten pieces with a rubber band around each set may be placed in the the tens column.

Uses for the place value chart may be the following:

1. Have pupils develop meaningful learnings pertaining to addition. If pupils, for example, are to add 25 + 16, five single congruent strips of paper may be placed in the ones pocket; two sets of ten each enclosed with a rubber band may be placed in the tens pocket. Thus, the numeral of 25 has been represented with two tens and five ones. Next, six congruent slips of paper may be placed into the ones pocket and a set of ten fastened with a rubber band placed in the tens packet. Thus, five ones and six ones may be joined together to represent one set of ten with a rubber band placed around this set and placed in the tens pocket. In the units pocket then there will be one member. In the tens pocket, there will now be two tens plus one ten plus one ten. The final sum is then 41, or four tens and one 1.

2. Guide pupils in attaching meaning to subtraction. If learners, for example, are to attach understanding to 35 - 16, five congruent slips of paper may be placed in the ones pocket. Three sets of ten with congruent slips in

each set may be put into the tens pocket (each set should have a rubber band around it). The pupil notices he cannot take six ones from five ones in the problem 35 - 16. He may take one set of ten from the tens pocket and place these into the ones pocket. The problem now results in the minuend containing two tens and fifteen ones. Now the pupil can take six ones from fifteen ones resulting in a difference of nine. Also, one ten taken away from two tens leaves one ten. Thus, 35 - 16 = 19.

3. Have pupils achieve basic understandings pertaining to multiplication. For example, if pupils are working on a problems such as 3 x 23, three congruent slips of paper may be placed into the ones pocket and two sets of ten each placed in the tens pocket. Next, two sets of three each may be placed into the ones pocket and two sets of two tens may be put into the tens pocket. Thus, pupils may understand that 3 x 3 = 9; this numeral would represent how many would be in the ones column. Also, three sets, two tens in each set, would make six tens in the tens column. The final product of 3 x 23 = 69.

4. Guide pupils in attaining desired learnings in division. If pupils, for example, are developing understandings pertaining to the division problem 42 ÷ 2, the dividend of 42 may be represented by two congruent strips of paper being placed in the ones pocket and four sets, with ten members in each set, being placed in the tens pocket. To divide the child may now place one member in each of two sets to represent the ones column. Next, the four sets of ten each on the tens pocket may be separated into two equal sets. Thus, the answer to the division problem 42 ÷ 2 is 21.

The teacher should utilize place value charts to help pupils attach meaning to understandings objectives in ongoing units of study in elementary school mathematics.

Using Transparencies and the Overhead Projector

Overhead projectors may wisely be utilized in the class setting due to the following inherent factors:

1. The teacher may face pupils when utilizing transparencies in a class discussion.
2. Specific content that pupils are to learn only, may be put on a transparency. Thus, irrelevant content is not a part of ongoing learning experiences.
3. Transparencies may be developed which are appealing and interesting to pupils.
4. The order of discussing content in several transparencies may be arranged sequentially from the point of view of the child's own unique perception.
5. Content may be added to any transparency as the need arises.

Transparencies and the overhead projector may be utilized in ways such as the following:

1. Pupils may count how many members there are in a set as given in a specific transparency.
2. Learners may tell how many members make a new set if two previously given sets are combined or joined together.
3. Pupils may tell how many members are left if, for example, there were nine circles and two were taken away.
4. Pupils may begin initial learnings in multiplication, e.g., three sets of circles with four members in each set, 3 x 4 =_____.
5. The inverse operation of multiplication (division) may be shown and discovered from the previous example, e.g., twelve circles are to be divided equally into three different sets. Thus, four members are in each of these three sets.

Using Filmstrips

There are leading publishers of filmstrips who have excellent curriculum materials for teachers to utilize in elementary school mathematics. These filmstrips should exemplify criteria such as the following:

1. The content must capture interests of pupils.
2. The diverse frames must follow appropriate sequential learnings for pupils.
3. Pupils should have ample opportunities to develop learnings inductively as well as deductively.
4. Problem-solving activities should be stressed adequately.
5. A manual should accompany the filmstrips to give possible suggestions as to their use.

Filmstrips may be utilized in learning experiences such as the following:

1. to introduces a new unit. If pupils, for example, are to study addition of unit fractions with unlike denominators, a carefully selected filmstrip may give learners an overview of the new unit. Pupils may then see how knowledge of unit fractions with unlike denominators can be useful in problem solving situations in class and in life outside of the school setting.

2. to develop learnings in greater depth. The content of filmstrips may guide learners in attaching meaning to fractions such as 1/4 + 1/6 = _____ or 1/3 + 1/6 = _____. Social situations would be stressed in the filmstrip presentation to guide learners in perceiving practical application of abstract learnings. In the filmstrip presentation, numerous experiences would be provided to pupils in understanding what is involved if unit fractions with unlike denominators are added.

3. to end or culminate a unit of study. In the filmstrip presentation, pupils should have ample opportunities to review what has been learned previously pertaining to adding unit fractions with unlike denominators. In reviewing what has been learned previously, pupils should have many opportunities to apply those learnings that have been developed previously. If pupils can use that which has

been learned previously within the framework of purposeful learning experiences, it will be possible to retain these understandings, skills, and attitudes for a longer period of time.

Content in the filmstrip should provide opportunities for pupils to

1. respond to questions and problems.
2. make practical application of what has been learned.
3. arrive at relevant concepts and generalization at their own unique rate of speed.
4. assess their own achievement in learning.
5. experience success and satisfaction in learning.
6. branch out in the direction of new related learnings.
7. achieve understanding of selected structural ideas in mathematics, e.g., commutative and associative properties of addition and multiplication.
8. perceive diverse operations in mathematics as being related, e.g. division undoes multiplication.

Using Graphs

Pupils in the elementary school should experience the making and using of graphs. Graphs developed by pupils with teacher guidance should

1. emphasize content within the experiences of learners.
2. present generalizations in a simplified manner.
3. contain a heading to orientate viewers to inherent conclusions.
4. provide interesting, meaningful, and purposeful learning experiences.

Kinds of graphs which may be developed by elementary pupils with teacher assistance include

1. picture graphs (Picture graphs may be the easiest to read and develop on the part of pupils).
2. line graphs.
3. bar graphs.

4. circle or pie graphs. (The teacher should develop and draw circle or pie graphs. Pupils can be guided in reading content from this kind or type of graph.).

Situations involving the use of graphs may include:

1. recording the number of visitors during Education Week. For example, if three visitors came to the class setting on Monday, three pictures in one to one correspondence can represent these guests. For Tuesday, if five visitors visited school, one picture should represent each of these guests individually in a picture graph. The graph should, of course, also show the visitors for Wednesday, Thursday, and Friday.

2. showing population figures of selected countries being studied in a social studies unit. The result could be recorded in a picture, line, or bar graph. Thus, if pupils are studying a unit on Common Market Countries of Western Europe, each country such as West Germany, Italy, France, and other members must be represented on the graph. Pupils with teacher direction should determine how many people will be represented by a picture on a picture graph. Or, in using line and bar graphs, the size of interval must be determined in terms of how many people will be represented therein. Each interval should be congruent in size. Thus, for example, in a bar graph, (horizontal or vertical bars) each one inch* bar may represent five million people. Pupils may then see that situations involving reality may be put in graphic form.

If pupils are studying a unit on Weather, a line graph may be developed pertaining to temperature readings for each of the days of the week during the time the unit is being taught.

Using Songs

There are numerous recordings which can help pupils to develop increased proficiency in rote counting. Learners in the early primary grade years generally enjoy singing selected songs. In learning to sing the song, "Ten Little Indians,"

*or one centimeter

pupils learn to use selected counting numbers in proper order or sequence. Thus, in singing "One little, two little, three little Indians; four little, five little, six little Indians; seven little, eight little, nine little Indians; ten little Indian boys," pupils are learning the correct order or sequence of counting numbers. Later on, pupils should attach meaning to rational counting; here, a learner points to an object and says "one", he points to a second object and says "two". Other sets, of course, may also be counted in sequence such as "three", "four", "five", and so on.

Pupils with teacher guidance may develop songs dealing with saying the counting numbers in proper sequence.

Using Money

Real coins and play money should be utilized in teaching-learning situations to help pupils understand not only money values but other relevant learnings. Thus, real and play money may be used in the following ways:

1. counting members given in a specific set, such as five coins pertaining to the cardinal number of five. Pupils may also develop learnings here pertaining to ordinal numbers, such as which coin is the third member in a specific set.

2. joining two sets together to make a new set, such as a child spending fifteen cents* for a candy bar and ten cents for a package of chewing gum. Thus, the learner can see practical application in the use of money when items are purchased.

3. subtracting when regrouping is involved such as a child having thirty-two cents and spending fifteen cents for a candy bar. The learner can be guided in understanding that a set of ten needs to be taken from the tens column in the value of three tens. These ten pennies may then be added to the two ones making a total of twelve cents. Now the pupil may take five ones from the twelve ones leaving seven cents. One ten may be taken from the two tens leaving one ten. The answer to the problem 32 - 15 is 17.

*Currency used depends upon nation involved.

4. multiplying a factor times a factor. If a pupil, for example, had 10¢ and needed four times that amount to buy candy, the learner could be guided to think about four distinct sets, each having ten members. Thus, 4 x 10 = 40.

5. dividing, such as twenty-five cents being divided among five boys. How many cents then would each receive?

In using real money as a learning activity in the mathematics curriculum, the following criteria should be followed:

1. Pupils should notice that money can buy needed goods and services.
2. Learners should apply what has been learned previously.
3. Pupils need to attach meaning to ongoing learning activities involving the use of money. Thus, pupils, for example, may count the number of coins in a set emphasizing the one to one correspondence concept. Learnings acquired by pupils must make sense and be comprehendable.
4. Learners need to be involved in determining learning experiences involving the use of money.

Using the Geoboard

In a modern program of elementary school mathematics, adequate emphasis must be given to the study of geometry. This would be true for the following reasons:

1. The world of geometry is all around us. Pupils may see squares (such as in tile on the floor), rectangles (such as in doors and window panes), circles (such as in circular windows and circle drives), and triangles.

2. Geometry can be interesting to pupils. Many learning experiences involving creative endeavors may be provided for pupils such as developing geometrical designs.

The geoboard made by the teacher or purchased commercially can provide interesting learning experiences for pupils. The base of a geoboard may consist of plywood. The size of the base can vary; eighteen inches by eighteen inches can provide a suitable size geoboard for teaching-learning situations. Small finish nails about a square inch apart may be driven into the geoboard. These nails should be driven in far enough to make for stability and yet protrude adequately to stretch around these protrusions using rubber bands to form triangles, squares, rectangles, and other geometric forms.

Using Drill and Practice in Mathematics

Once pupils understand in a meaningful, purposeful, and interesting manner basic addition, subtraction, multiplication, and division facts, drill procedures may be utilized in teaching-learning situations. Varied procedures must be used to provide drill sessions for pupils pertaining to the basic facts. The interests of pupils must be developed and maintained in the elementary school mathematics curriculum. The following procedures may be utilized to fix basic addition, subtraction, multiplication, and division facts in the mind of the learner:

1. flash cards(pupils are given the time they need to respond to facts such as the following, each on a three by five-inch card):

$$\begin{array}{rrrrrrrrr} 3 & 5 & 4 & 4 & 5 & 6 & 3 & 2 & 6 \\ \underline{+5} & \underline{+3} & \underline{+4} & \underline{+5} & \underline{+4} & \underline{+3} & \underline{+6} & \underline{+6} & \underline{+2} \end{array}$$

2. games (these may be made by the teacher); pupils move forward a certain number of squares on a board if they respond correctly to a basic addition, subtraction, multiplication, or division fact. The fact to be responded to would be on a small card, face down, drawn in the order of players taking their turns playing the game. Penalties such as losing a turn or moving back a space or more may be inherent in the game.

3. fishing for fish (small "fish" may be cut from different colors of construction paper--each fish has a paper clip in its mouth.) The child uses a "fishing pole" consisting of a stick, attached string, and a magnet to catch fish; each fish has a number pair printed on it such as

$$\begin{array}{ccccccc} 5 & 4 & 3 & 5 & 6 & 6 & 7 \\ \underline{-3} & \underline{-3} & \underline{-2} & \underline{-4} & \underline{-3} & \underline{-2} & \underline{-5} \end{array}$$

A child may keep the fish caught if he gives the correct answer to the number pair.

4. slides and transparencies. The teacher may write basic number pairs on individual slides. Pupils can then respond to these number pairs by attempting to give the correct answer. Basic number pairs may also be written on transparencies: pupils may then given correct answers to these problems.

Practice would involve guiding pupils to use previously developed learnings in new situations. Learners must be aided to transfer learnings obtained from one situation to the next problematic area. Thus, if pupils have learned that 10 + 5 = 15 in a meaningful, purposeful, and interesting manner, they should be able to apply these learnings to a new situation such as determining the cost of items that costs 10 cents and 5 cents, respectively. Thus, facts, concepts, main ideas, and generalizations that have been acquired may be utilized in function situations. Content learned previously that is used in functional everyday life situations generally is not forgotten by pupils.

The following criteria should be followed by teachers of mathematics when teaching for a transfer of learning:

1. Have pupils attach meaning to new learnings being acquired.
2. Point out to learners or have them discover how previously attained learnings may be used in new situations.
3. Have pupils perceive reasons or purpose for learning content in elementary school mathematics.
4. Guide pupils in perceiving essential understandings when learning a new process.

In Summary

Teachers need to provide a variety of learning experiences for pupils in elementary school mathematics. This is necessary due to pupils

(a) achieving at diverse levels of accomplishment in the mathematics curriculum.

(b) individually possessing different learning styles.

The following, among others, can be relevant learning activities to present to pupils:

1. using a selected series fo elementary school mathematics textbooks.
2. utilizing the flannel board to guide individual pupil achievement in mathematics.
3. Helping pupils attach meaning to learning through the use of markers.
4. guiding pupils in learning by using place value charts.
5. aiding learner achievement through the use of transparencies and the overhead projector.
6. stimulating learner interest in mathematics with the use of selected filmstrips.
7. using graphs in functional situations.
8. helping young pupils to develop interest in numbers by singing songs directly related to ongoing units of study in elementary school mathematics.
9. using the geoboard to help pupils experience the world of geometry.
10. providing drill and practice for pupils so that previous developed learnings will not be forgotten.

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PROVIDING FOR INDIVIDUAL DIFFERENCES AND DESIGNING THE MATHEMATICS CURRICULUM

Part One: Providing For Individual Differences

Teacher, principals, and supervisors need to become thoroughly familiar with individual differences among pupils and how to guide each learner to achieve optimal development. Thus, it is important to be highly knowledgeable about traits pertaining to slow learners, average achievers, as well as talented and gifted pupils.

The Slow Learner

Which traits and characteristics do slow learners possess? Generally, it can be said that slow learners may be described in the following ways:

1. They may come in the category of having IQ's or Intelligence Quotients ranging from 75-90.

2. Their achievement is lower than that of average achievers and will register lower than their present grade level average expectancy.

3. The attention span of slow learners is shorter than that of higher achievers.

4. It takes more time for the slow learner to understand a new process in mathematics as well as to master basic addition, subtraction, multiplication, and division facts.

5. These learners need more of concrete and semi-concrete learning activities than do faster learners.

6. Slow learners need learning activities which provide proper sequence.

7. More supervision and direction may need to be given slow learners compared to pupils who achieve at a higher level.

8. A lack of opportunities to learn in the home setting may be the lot of

many slow learners.

9. It is necessary for teachers to be patient and understanding in teaching pupils achieving at a slower rate of speed as compared to faster achievers.

10. Slow learners generally ask fewer questions and may reveal less curiosity in the school and class environment as compared to peers of similar age levels.

Talented Pupils

Teacher, principals, and supervisors need to identify and provide adequately for talented learners in the class setting. Talented pupils may reveal characteristics such as the following:

1. having a rather lengthy attention span.
2. possessing the ability to gain understandings and skills quickly.
3. being able to retain learnings well.
4. completing tasks more rapidly than peers in the class setting.
5. showing much curiosity in learning.
6. revealing tendencies of being creative individuals.
7. becoming more independent in learning.
8. having a desire to complete additional work in the school and class setting.
9. possessing knowledge and skills helpful in the areas of problem solving.
10. being able to engage in constructive learning activities when spare time is available in the school and class setting.

Comparing Talented Pupils and Slow Learners

There are always exceptions to statements made about human beings. However, in general the following comparisons may be made between talented pupils and slow learners:

1. Talented pupils retain learnings longer than do slow learners.
2. Talented learners possess more initiative and become more independent

in learning as compared to peers in the school and class setting.

3. Pupils with much ability acquire learnings sooner as compared to learners with less capacity.

Learning Activities and Individual Differences

The teacher must select interesting, meaningful, and purposeful learning experiences to provide for individual differences in ongoing units of study in mathematics. Slow learners will need the following kinds of experiences:

1. Concrete phases of instruction need to be emphasized adequately whereby pupils use bean and corn seeds, checkers, crayons, beads, sticks, and other objects to clarify learnings in counting, addition, subtraction, multiplication, and division.

2. the teacher must explain new processes in mathematics thoroughly and patiently to slow learners. Adequate opportunities, of course, must also be given to aid these pupils in learning inductively.

3. Adequate emphasis must be given to help slow learners develop sequential learnings. Too frequently, the teacher wants to "jump" too far ahead of these pupils in teaching-learning situations. Thus, selected slow learners have been taught addition involving regrouping and renaming (e.g., 48 + 17 = _____; 39 + 18 = _____) whereas these learners need more practice and guidance in simple addition (e.g., 32 + 13 = _____; 43 + 13 = _____).

4. The teacher needs to observe the attention span of slow learners in each learning activity. Thus, slow learners need to experience a new learning activity in mathematics before the preceding experiences become dull and boring.

5. Adequate supervision and guidance must be given to slow learners to aid them in achieving optimal development in mathematics. Thus, the teacher needs to evaluate if these pupils have achieved a desired objective before the next sequential objective is stressed in a new lesson or unit of study. Kindergarten and first grade pupils, for example, should be able to count to ten in a rote manner before counting the number of members in a set of ten.

6. There are many abstract symbols which may be emphasized in teaching-learning situations in ongoing units of study in mathematics. These abstract symbols include + (plus), x (times), ÷ (divided by), = (equals), and others. Pupils should be guided to attach meaning to these symbols in functional situations. Abstract symbols in mathematics should not be emphasized in ongoing units of study to the point where slow learners feel frustrated and lack feelings of success in the mathematics curriculum.

7. Slow learners may be less creative in finding diverse solutions to a problem as compared to faster learners in the class setting. Thus, for example, a slow learner may be able to find one solution to a problem such as

$$\begin{array}{r} 42 \\ \underline{x6} \\ 12 \\ \underline{240} \\ 252 \end{array}$$

Other algorisms could include the following:

(a) $$\begin{array}{r} 42 \\ \underline{x6} \\ 240 \\ \underline{12} \\ 252 \end{array}$$

(b) $$\begin{array}{r} 42 \\ \underline{x6} \\ 252 \end{array}$$

8. Slow learners need ample opportunities to engage in practice and drill activities involving previous learnings obtained from relevant units of study. Thus, slow learners need to experience new learning activities directly related to content mastered previously. If a pupil, for example, has learned in a meaningful, interesting, and purposeful way that 8 + 4= 12 and 4 + 8= 12, he/she may practice related learnings in which these addition facts are contained in relevant word problems. Drill as a method of teaching could involve the use of flash cards, games, and the overhead projector to fix specific learnings in the minds of pupils. Thus, a pupil may need drill with the use of flash cards pertaining to the following:

$$\begin{array}{cccc} 8 & 5 & 13 & 13 \\ \underline{+5} & \underline{+8} & \underline{-8} & \underline{-5} \\ 13 & 13 & 5 & 8 \end{array}$$

9. Slow learners need to experience objectives in mathematics which are attainable. Pretesting of slow learners before a new unit in mathematics is

implemented aids in adjusting the new unit to the present achievement levels of these learners. Objectives for slow learners to attain should not be too difficult nor should they pertain to what these learners have already acquired. Thus, new learnings may be gained by slow learners when achieving desired objectives in mathematics and at the same time success can be inherent in these experiences.

10. Slow learners should be guided to develop feelings of an adequate self-concept. These learners too frequently have experienced failure in mathematics, as well as other curriculum areas. Other pupils in the school and class setting may have minimized the worth and achievement of the slow learner. All pupils in the school and class setting need to be respected for their intrinsic worth regardless of ability, socio-economic level, or creed.

Adequate provision needs to be made for the fast learner in the mathematics curriculum. These pupils need to experience the following:

1. There needs to be less emphasis upon the concrete stage of learning and more emphasis upon abstract learnings. The teacher, however, must observe if these fast students are ready for relevant abstract learnings and guide each pupil to acquire optimal development.

2. The teacher generally needs to give less of explanations to these learners in terms of understanding new concepts and processes in the mathematics curriculum. Talented learners can become quite self-directed and independent in their work.

3. Fast learners can gain mathematical concepts and generalizations readily. These learners with teacher guidance may become relatively independent in sequencing their own learnings.

4. The talented pupil has a longer attention span as compared to the slow learner. Thus, time devoted to the teaching of mathematics should harmonize with the attention span of these learners and result in a balanced program of learning for pupils in the elementary school.

5. Less direct supervision in mathematics of talented and gifted pupils needs to be in evidence by the classroom teacher. These learners in the class setting can be independent and responsible in ongoing units of study in mathematics.

6. Talented and gifted pupils may be highly fascinated and challenged when encountering abstract symbols and concepts in the mathematics curriculum. A stimulating environment with interesting, mearningful, and purposeful learning experiences will aid pupils in wanting to acquire relevant abstract content. Thus, for example, talented first grade pupils may attach meaning to abstract concepts, such as lines, line segments, points, and rays. With appropriate sequence in learning talented sixth graders, as a further example, can attach meaning to abstract concepts such as square root, complex numbers, irrational real numbers, and tolerance.

7. Talented and gifted pupils in the mathematics curriculum should be guided to become increasingly independent in their work. Thus, these learners should have ample opportunities to select what to learn, as well as methods of learning. With teacher guidance, these pupils should be guided in evaluating their own achievement in ongoing units of study as well as with a specific problem or area of difficulty being experienced. Talented and gifted pupils with teacher leadership should assess their own progress in units of study dealing with the following:

(a) sets and mathematical sentences.

(b) structural ideas or properties in mathematics.

(c) basic addition, subtraction, multiplication, and division facts.

(d) algorisms for addition, subtraction, multiplication, and division using whole numbers.

(e) prime numbers, composite numbers, and integers.

(f) addition, subtraction, multiplication, and division of fractional numbers.

(g) addition, subtraction, multiplication, and division using decimals and per cents.

utilized to provide for slow learners, average achievers, as well as talented and gifted pupils:

1. using behaviorally stated objectives. Learners of different achievement levels would achieve the same measurable objectives. However, slow learners attain each objective in sequence at a slower rate as compared to average achievers and fast learners.

2. pretesting pupils to ascertain their own individual achievement levels at the beginning of a new school year when using a series of reputable mathematics textbooks. Each pupil depending on his/her present achievement level would be participating in learning activities directly related to pretest results based on content from a series of mathematics textbooks. Thus, for example, a fourth grade pupil may be participating in learnin experien involving the use of a third grade mathematics textbook, while a talented and gifted peer may be involved in learning experiences using content from a fifth or sixth grade mathematics textbook.

3. providing learning centers to enrich experiences for talented and gifted pupils who have completed assigned work in ongoing units of study in mathematics.

4. grouping pupils within the class setting to provide for individual differences. Thus, slow learners, average achievers, and fast learners could be taught in separate groups in an atmosphere of respect. The mathematics curriculum would be adjusted to the present achievement level of each of these groups of learners.

5. grouping pupils homogeneously for mathematics instruction on the intermediate grade levels. Thus, if feasible, fast learners could be taught in one class setting, average achievers in a different class setting, with slow learners comprising a third group.

6. using a variety of materials and methods in the mathematics curriculum. Thus, the mathematics curriculum is adjusted to diverse learning styles exhibited by learners. Also, there are more complex learnings in mathematics available for fast learners as compared to other levels of achievement. With the use of concrete

(h) metric and nonmetric geometry.

(i) other systems of numeration.

(j) uses for probability and statistics.

(k) functions in the mathematics curriculum.

8. Talented and gifted pupils should be guided to achieve to their optimum in individual endeavors as well as in committee work in the class setting. The teacher must guide these learners to experience continuous progress in ongoing units of study in mathematics as well as to work together harmoniously with others. Too frequently, the mathematics teacher may emphasize individual efforts on the part of the pupil largely. Thus, committee work on the part of pupils is greatly deemphasized. Certainly, there needs to be rational balance between pupils working on an individual basis as well as within a committee setting. Gifted and talented pupils may work together within a committee to understand a new process, check computations, and/or solve a problem in an ongoing unit of study in mathematics. Talented and gifted pupils could also assist slower achievers in the mathematics curriculum. There are several values inherent in a learning activity of this kind involving fast learners helping slow learners in mathematics units of study:

(a) Talented and gifted pupils may develop feelings of an adequate self by helping others who need assistance.

(b) Fast learners may understand previously developed learnings more fully by explaining related content to pupils who learn at a slower rate.

(c) Talented and gifted pupils may develop wholesome attitudes toward those who experience difficulty in learning.

Talented and gifted pupils as well as those learners who achieve at a slower rate need to develop feelings of being accepted by others, belonging to a group, having security and status in the school and class setting, and being able to participate freely in ongoing learning experiences.

Specific Ways of Providing for Individual Differences

There are numerous ways available to provide for individual differences in the mathematics curriculum. The following approaches, among others, can be materials (real objects or replicas), differences between and among pupils in mathematics achievement may be more adequately provided for as compared to the use of abstract learnings. Thus, in using a meter stick (a concrete object) in a unit on "Measurement and the Metric System," slow learners may measure the height of selected objects such as a door, window, and desk. Talented learners may use a meter stick to find the area of the classroom or the volume of selected containers.

When abstract learnings are to be acquired in mathematics, it is much more difficult to provide for individual differences. For example, if pupils are to learn to divide the following as a new experience for all learners in a heterogeneously grouped classroom; 8478 ÷ 36, talented pupils will generally develop new understandings much more rapidly as compared to peers of similar chronological age. Thus, if talented pupils are ready for developing learnings pertaining to the previously named division problem slow learners and average achievers may then not possess needed background learnings.

7. accelerating achievement of talented and gifted pupils in the mathematics curriuclum. In a nongraded program of mathematics instruction, fast learners can be guided to experience continuous progress within the framework of a challenging and meaningful curriculum. Thus, for example, in a fourth grade class, selected talented learners may be acquiring relevant experiences based on criteria related to fifth, sixth, or seventh grade level of attainment.

8. organizing a mathematics club to provide for individual differences. Selected pupils with teacher guidance may wish to organize a mathematics club as a means of enriching the curriculum as well as providing for individual differences. Thus, at selected regularly scheduled intervals, meetings may be held involving members in the mathematics club. The members may

(a) view and discuss content from filmstrips relating to stimulating topics in mathematics.

(b) identify topics, units, and specific problems in mathematics for discussion and evaluation.

(c) interact with a resource person (high school or university mathematics instructor) involving a challenging area of interest in mathematics.

(d) make models pertaining to learning gained from ongoing units of study in geometry.

In Summary

Teachers, principals, and supervisors need to study, appraise, and implement research findings pertaining to helping slow, average, and fast achievers realize optimal development in the school and class setting. Thus, educators in the public schools need to ask questions and attempt to arrive at solutions pertaining to the following problem areas:

1. How do slow learners differ from average achievers in intellectual, emotional, social, and physical development? How are slow learners different from fast learners in these same four facets of development?

2. How can educational objectives be selected which are attainable for slow, average, and fast achievers?

3. What criteria should be utilized to select learning activities for all pupils in the school and class setting so that each may experience continuous progress in the mathematics curriculum?

4. How can each child's progress in different curriculum areas be evaluated appropriately and thus help to insure the best quality learning experiences possible for each individual learner?

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Part Two: Designing the Mathematics Curriculum

Teachers, principals, and supervisors need to study, evaluate and implement in the public school setting selected accepted ideas pertaining to an effective design in the elementary school mathematics curriculum. Thus, concepts such as scope and sequence become important when an appropriate design is developed for a relevant elementary school mathematics curriculum.

Scope in the Mathematics Curriculum

The question frequently arises as to "what should be taught in elementary school mathematics." Which understandings, skills, and attitudes should pupils develop to become proficient and contributing members in American society? There are no clear-cut, easy answers to this quesion. There are, however, selected individuals and groups of individuals whose studies and/or statements of thought and philosophy may provide relevant guidelines.

1. Writers of reputable mathematics textbooks can provide a guide in determining "what" (scope) pupils are to learn in an up-to-date curriculum. There are advantages as well as disadvantages in using this approach rather exclusively. Advantages could be the following:

(a) Recognized writers and involved publishing companies in the field have spent much time and money in developing and evaluating content contained in mathematics textbooks for pupils.

(b) The teachers' manual related to the elementary school mathematics textbook for pupils can have excellent teaching suggestions to utilize in ongoing units of study.

(c) The textbook for pupils on any grade level clearly indicates the inherent unit titles for learner interaction in teaching-learning situations.

(d) Learning activities for pupils within each unit of study are clear and sequentially presented.

(e) The contents of the textbook can be utilized to provide for individual differences in the class setting by having pupils work at different present achievement levels.

(f) Much of the work of the teacher has been done in developing the mathematics curriculum when utilizing a reputable mathematics textbook to select objectives, learning experiences, and appraisal techniques in ongoing units of study.

(g) Pupil interest and purpose can be developed within specific units of study when effectively utilizing a relevant series of elementary school mathematics textbooks. Carefully selected experiences for pupils can aid in stimulating interest and purpose for learning.

Disadvantages in using mathematics textbooks almost exclusively would be the following:

(a) Sameness in kinds of learning experiences provided for pupils may become rather boring to learners. Pupils generally like to experience a variety of learning experiences including theuse of reputable elementary school mathematics textbooks.

(b) The heavy use of textbooks in the mathematics curriculum may not meet the needs, interests, and abilities of selected pupils.

(c) Pupils differ from each other in terms of learning styles possessed. Thus, the learning style of a few pupils may not harmonize with the heavy use of textbooks in providing content for the mathematics curriculum.

(d) Teachers need to be creative in selecting objectives, learning activities, and appraisal techniques in ongoing units of study. Heavy use of mathematics textbooks in teaching-learning situations can deemphasize creative efforts of teachers.

(e) There is a tendency to keep an entire class of pupils together at the same place at the same time when textbooks are emphasized heavily in teaching-learning situations. Thus, individual differences are not adequately provided for in the class setting.

(f) Units of study in a series of mathematics textbooks may overemphasize selected facts, concepts, and generalizations and deemphasize other relevant learnings for pupils. For example, the structure of mathematics may be overly stressed to the point where computation skills are deemphasized.

It appears desirable to utilize reputable elementary school mathematics textbooks along with other carefully chosen learning activities to provide for the needs, interests, and abilities of individual learners.

Emphasis upon the use of selected elementary school mathematics textbooks to determine the scope (what is taught) in ongoing units of study for a specific school year may reveal the following content, for example, for the fifth grade level:

(a) Sets, Numerations, and Number.

(b) Properties and Patterns of Addition and Subtraction

(c) Properties of Multiplication and Division.

(d) Fractional Numbers and Their Operations.

(e) Decimals, Ratio, and Proportion.

(f) Nonmetric Geometry.

(g) The Metric System.

(h) Metric Geometry--Finding Perimeter, Area, and Volume.

(i) Measurement in Today's Society.

(j) Line, Bar, and Picture Graphs; Probability.

(k) Numeration Systems (other than base ten).

Thus, in utilizing a reputable elementary school mathematics textbook as a basis for providing content in teaching/learning situations, it appears that the following would be viable criteria to follow:

(a) Use a variety of learning activities--such as markers of different kinds, films, filmstrips, transparencies and the overhead projector--to enrich experiences of pupils and to provide for individual differences.

(b) Preassess pupils to determine where each should begin at the beginning of a specific school year within a given unit of study, and guide each learner to experience continuous progress.

(c) The scope of the mathematics curriculum should include those learning experiences which will guide learners to become contributing members in society.

2. The teacher may write measurable behaviorally stated objectives when determining scope in the elementary school mathematics curriculum.

The following are examples of behaviorally stated objectives in a unit on "Addition and Subtraction of Whole Numbers."

(a) The pupil will add correctly nine of ten problems; each addition problem contains two one-digit addends.

(b) The pupil will subtract correctly nine out of ten problems containing a one-digit minuend and one-digit subtrahend.

(c) The pupil will solve correctly five out of six word problems containing two one-digit addends to be added.

(d) The pupil will subtract correctly five out of six word problems containing a one-digit minuend and a one-digit subtrahend.

For each of the above behaviorally stated objectives, the teacher needs to select interesting, purposeful, and meaningful learning experiences in order that learners may achieve these desired ends sequentially. Pupil achievement may then be measured if each objective has or has not been attained. If an objective is not realized by a learner, the teacher would need to diagnose to determine the cause or causes for this happening. Additional learning experiences then need to be provided for these learners so that they may also be successful in attaining stated measurable objectives.

Thus, behaviorally stated objectives written by the teacher (or teachers) pertaining to diverse units of study in mathematics can pertain to what is taught or the scope of that curriculum area.

3. Pupils should achieve structural ideas in elementary school mathematics. Jerome Bruner, psychologist from Harvard University, has been very instrumental in stressing the importance of pupils perceiving properties or the structure of a curriculum area. Pupils would continually understand these structural ideas in greater depth as they progress through the public school years. Jerome Bruner in the book, The Process of Education, writes the following:

> We begin with the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development (page 33).

Important structural ideas that pupils may achieve in mathematics could include the following:

(a) the commutative property of addition and multiplication.

(b) the associative property of addition and multiplication.

(c) the distributive property of multiplication over addition.

(d) the identity elements for addition and multiplication.

(e) the property of closure for addition and multiplication.

Jerome Bruner would stress the importance of pupils developing structural ideas inductively. Thus, the teacher must

(a.) select relevant learning experiences which would guide pupils to discover structural ideas in mathematics. (The teacher definitely would not lecture or present long explanations to pupils on the meaning or meanings attached to these key ideas).

(b.) assist learners to continually discover these structural ideas as they progress through the public school years. (These key ideas cannot be mastered on any one grade level in the elementary school)

(c.) guide pupils to realize these structural or key ideas on increased levels of complexity as learners achieve continuous progress in the school and class setting. (For example, on the first grade level, most pupils learn that "5+7=7+5." On higher grade levels pupils learn that "13+18=18+13; 145+258=258+145; 3296+1835=1835+3296," and so on ---the commutative property of addition.)

Reasons for emphasizing the structure of knowledge in teaching pupils include the following:

(a) Subject matter specialists can do a better job of selecting what is important for pupils to learn as compared to teachers, principals, and supervisors in the class setting.

(b) It is the teacher's role to choose learning experiences which will guide pupils to achieve inductively the identified structural or key ideas as determined by content specialists.

(c) Subject matter specialists and educators must work together to improve the public school curriculum.

(d) Public school pupils should utilize methods of study emphasized by subject matter specialists such as that used by mathematicians.

4. Pupils may experience the use of programmed learning in the mathematics curriculum.

Programmed learning follows selected ideas pertaining to how pupils learn. The following are major generalizations pertaining to programmed learning:

(a) Pupils progress in very small steps in learning.

(b) Each small step of learning is sequentially arranged.

(c) In the use of programmed materials, the pupil generally looks at a picture, reads related content, responds to an item, and then checks the correctness of the response. The first step of learning indicated above (looking at a picture) may be omitted in some programmed materials. The sequence of these steps in learning is repeated again and again as far as pupil learning is concerned.

(d) The learner knows almost immediately if he/she is right or wrong in terms of responses given. If the pupil was correct in the response given, reinforcement in learning is involved. If an incorrect response was given by the learner, he/she now knows the correct answer after checking with the correct answer indicated by the programmer.

(e) The pupil may progress at his own optimal rate of learning when programmed materials are used.

(f) Rarely does a learner respond incorrectly to an item in sequence since the progressive steps in learning are small enough generally to prevent incorrect responses.

Disadvantages in utilizing programmed materials include the following:

(a) Problem solving is generally not stressed in programmed materials.

(b) The sequential steps in learning may be too small for certain pupils. Gifted and talented students may, of course, not need these small sequential steps in learning to be successful achievers.

(c) If programmed materials were used exclusively in learning, boredom may set in on the part of individual learners. Human beings seemingly crave a variety of experiences in the school and class setting as well as in life.

(d) The learning styles of selected pupils may not harmonize with the philosphy inherent in programmed learning in the class setting.

Programmed materials emphasize the Stimulus-Response school of thought in terms of how pupils learn. Thus, pupils in the mathematics curriculum using programmed learning materials must experience a Stimulus, such as seeing a picture of three boys and being asked to tell how many members are in this set. The pupil gives a Response to this item by writing the the numeral "3" or the word "three." Next, the learner may check the correctness of his response with the answer given by the programmer in the programmed mathematics textbook. Similar, small sequential steps in learning then follow for the individual pupil utilizing programmed materials.

The teacher of mathematics could program content for pupils pertaining to arithmetic, algebra, and geometry. These resulting materials must

(a) present learnings in small steps to pupils (Stimulus)

(b) give pupils an opportunity to respond to each item (Response)

(c) be arranged sequentially to insure pupil success in responding to each item.

(d) give learners an opportunity to check the correctness of each response made before going on to the next sequentially developed item.

5. Pupils should experience problem-solving activities in the class and school setting. The Gestalt school of thought in terms of how pupils learn would emphasize the following conditions for pupils:

(a) The whole child is involved in learning such as the emotional, intellectual, social, and physical facets of an individual's development.

(b) The pupil perceives situations as wholes first rather than parts.

(c) The wholeness of a situation can be analyzed in terms of parts.

Pupils in a stimulating learning environment should have ample opportunities to identify problem areas. Learners, for example, may identify problem areas in the mathematics curriculum from the following learning activities:

(a) Viewing a bulletin board display emphasizing four cookies to be divided among eight pupils. Division of a counting number by a counting number resulting in a fraction may be emphasized in an ongoing unit of study. A pupil may think of related situations in life that he/she is facing, a situation whereby a counting number is to be divided by a counting number resulting in a fractional value.

(b) Viewing a filmstrip on sets. Thus, the pupil is noticing in this presentation how two sets which are disjoint may be joined together to form a new set. An involved pupil may have wanted to find out how many marbles he/she now has after originally having a set of eight, and six additional marbles were added to the set as a gift from a peer.

(c) Watching a demonstration by the teacher using markers to show the meaning of a subtraction problem involving a single digit minuend and a single digit subtrahend. The involved learner now thinks of a related problem in subtraction which needs solving as far as his/her own personal experiences in everyday living are concerned.

(d) Noticing situations in the class setting which require problem-solving skills. Thus, from the class and school environment, pupils notice a problem or problems which need solving pertaining to the use of arithmetic, algebra, and geometry. Pupils may then measure, weigh, or find the volume of a container in a real life situation. For example, selected pupils may need to find how many cups* are in a pint when following directions in developing a food product in the class setting.

In daily situations outside the school and class setting, pupils, of course, detect problems needing solutions involving the use of mathematics:

An eight year old has saved $6.75** from allownace money received, he desires to buy a baseball glove costing $12.98. The child attempts to determine how much more money is needed to make the purchase. The eight year old thinks about different approaches to use in coming up with a solution to the problem. The child brings to bear related knowledge in attempting to solve the problem. Ultimately, the eight year old may count by fives or tens from $6.75 to $12.98 to determine

*can substitute metric measurements
**use currency of nation involved

how much additional money will be needed to buy the baseball glove.

The Gestalt approach in learning emphasizes

(a) wholistic situations in life from which a problem or problem areas are identified.

(b) previously gained related knowledge is brought to bear upon the solution of the problem.

(c) insight is gained by individuals in obtaining a needed solution.

(d) solutions to problems are held tentative. Thus, new problems are identified from existing situations.

In Summary

There are numerous schools of thought in psychology and philosphy attempting to explain how individuals learn. Teachers, principals, and supervisors must study and appraise diverse schools of thought in education pertaining to how human beings gain learnings. Ultimately, educators in the public school setting must develop a philosophy and psychology of their own to implement in teaching-learning situations. Teachers, principals, and supervisors may develop a rather consistent school of thought such as Stimulus-Response or the Gestalt approach. Other public school educators may be eclectic in teaching-learning situations selecting that which is deemed relevant and viable from diverse schools of thought explaining how individuals learn.

Whichever school of thought is selected, the following principles of learning should be accepted by all teachers, principals, and supervisors in the school setting pertaining to teaching mathematics and other curriculum areas:

(1.) Learning activities must capture the interests of pupils.

(2.) Pupils must understand what is learned.

(3.) Balance among understandings, skills, and attitudinal goals must be emphasized in ongoing units of study.

(4.) Good sequence in learning must be inherent on the part of learners.

(5.) Pupils need to feel successful in learning.

(6.) Learners must develop feelings of an adequate self-concept.

(7.) Each pupil must experience adequate readiness activities prior to experiencing new learnings.

(8.) A variey of learning experiences must be provided for learners.

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PRINCIPLES OF LEARNING AND THE MATHEMATICS CURRICULUM

There are selected principles of learning that need adequate emphasis in teaching-learning situations in the mathematics curriculum. These principles or guidelines should aid pupils in attaining optimal achievement. Which principles of learning should the teacher stress in ongoing units of study?

Purpose in Learning

Pupils need to perceive purpose or intent to learn. Thus, if learners, for example, are studying a unit on "Uses of Graphs," adequate time should be taken to develop reasons for participating in ongoing learning activities. Pupils could be guided in developing a picture graph pertaining to visitors to the school-class setting during American Education Week or National School Lunch Week. Experiences that pupils have personally in these situations can be shown on the picture graph. A cutout or a drawing for each visitor can be put in the picture graph for each of the days of the week--Monday through Friday.

Pupils may be guided to notice that the contents of a picture graph can simplify information for readers. Thus, the reader of the completed graph may quickly notice trends in terms of visitors, for each of the days of the week, coming to the school-class setting.

Sequentially, as pupils progress through the public school years, more complex learnings may be stressed in developing diverse kinds of graphs such as in the following learning activities:

1. Showing population figures of diverse countries being studied in ongoing social studies units on picture, line and bar graphs.
2. Presenting data pertaining to growth in the Gross National Product (GNP) covering several decades within the framework of appropriate kinds of graphs.
3. Using relevant graphs portraying data on inflation covering selected years.

Intent to learn or reasons for attaining selected understandings and skills need to be emphasized when initiating a unit as well as when developmental and culminating activities are in evidence. Appropriate attitudes may be a significant end result when pupils perceive purpose in learning.

Interest in Learning

Interesting learning experiences need to be provided for pupils. Thus, learners may attend to and extract relevant information as well as abilities from ongoing activities. There are several methods to utilize in emphasizing the concept of interest in the mathematics curriculum. The teacher, for example, may utilize a variety of activities such as markers, place value charts, an abacus, filmstrips, slides, films, pictures, as well as content from reputable mathematics textbooks to stimulate pupil interest in learning. Varying learning experiences are necessary to develop and maintain interest in ongoing units of study. Attempts also need to be made to determine present achievement levels of each learner in the mathematics curriculum. Continuous progress then is possible when new objectives, related learning activities, and evaluation procedures are sequentially perceived by learners. Pupils may lose interest in learning if content to be learned is excessively complex or easy. Learning experiences need to be challenging but not overwhelming!

Problem Solving in the Mathematics Curriculum

Learners need to have ample opportunities to engage in solving realistic problems. Situations in life demand that human beings become proficient in problem solving. Thus, pupils should have ample opportunities to engage in the solving of real problems. Pupils with adequate background knowledge could solve problems such as the following:

1. A miniature supermarket could be housed in the class setting. Learners may bring empty cereal boxes, fruit and vegetable containers, candy bar wrappers, flour sacks, and sugar bags. These items should be placed on a counter, properly labeled and priced. Pupils may "buy" needed items using toy money. Thus, needed addition, subtraction, multiplication, and division facts may be learned in this manner.

2. A "cafeteria" could also be set up in the class setting. Cutouts of appropriate food items may be pasted on paper plates. Each food item would need to be priced meaningfully. Learners again may use toy money to purchase selected food items in the "cafeteria."

3. The mathematics laboratory concept of teaching and learning can well become an important facet of the mathematics curriculum. Thus, pupils may measure areas, distances, and determine volumes of specific containers in actual problem solving situations utilizing the English as well metric systems of measurement.

4. Realistic problems may also be solved by pupils within the framework of the use of reputable textbooks, films, filmstrips, slides, video-tapes, and life-like situations in society.

Meaningful Learnings in the Curriculum

Pupils need to understand and attach meaning to learnings obtained in ongoing units of study. For learnings to be meaningful to pupils, the following criteria may well need to be in evidence:

1. Adequate emphasis placed upon manipulative materials, and semi-concrete materials before emphasizing abstract learnings in the mathematics curriculum.

2. Sequential experiences perceived by learners need to be inherent in teaching-learning situations.

3. Adequate readiness experiences to progress to increasingly more abstract levels of learning.

4. Pupil-teacher planning being a part of the mathematics curriculum.

If objectives are excessively easy to attain, selected learners generally will feel a lack of challenge in learning. Also, if the objectives in ongoing units of study are excessively complex, pupils may not have needed background experiences to make adequate progress. Thus, for each learner, new objectives to achieve need to be in evidence; however, pupils individually need to be successful in their attainment.

Providing for Individual Differences

There are diverse ways to provide for individual differences in achievement in the mathematics curriculum. The following, among others, are ways to provide for diverse achievement levels in the mathematics curriculum:

1. Use the mathematics laboratory concept in teaching-learning situations. Thus, pupils on an individual bases sequence their own progress through the actual weighing of selected items, measuring of surfaces, as well as finding the volume of selected containers.

2. Utilize learning centers in the school-class setting. Learners sequentially choose the task to work on at a particular center. Ideally, the task or learning activity provides for new challenging experiences.

3. Utilize problem solving methods. Pupils with teacher guidance may select realistic problems to solve on an individual basis. These problems must be on the present achievement levels of individual learners.

4. Pretest pupils using a reputable series of mathematics textbooks. Each learner is then at a different place within the confines of the textbook in terms of achievement. Pupils individually progress as rapidly as possible in satisfactorily completing sequential learnings in the textbook. The teacher gives explanations and guidance to learners as the need arises. Continual help is also given to diagnose errors made by learners in specific problematic situations. Remedial aid is given to learners to overcome identified deficiencies.

5. Use contracts in the mathematics curriculum. Pupils with teacher guidance write up in contract form what the former are to achieve within a given period of time, such as a few days or a week. The level of accuracy of completed work may also be spelled out in the contract. Both pupil and teacher sign the agreement or contract. If the contents of the contract later appear too difficult for the learner to achieve, needed modifications can then be made.

6. Have pupils individually achieve objectives of diverse levels of achievement. Thus, for example, pupils who achieve at a less complex level, as compared to fast achievers, may be guided in attaining objectives suitable to their optimal level of development.

The teacher needs to think of and implement the principle of providing for each level of achievement in the mathematics curriculum. Only then, can learners attain optimal achievement.

In Summary

There are selected principles of learning which need adequate emphasis in the mathematics curriculum. These include:

1. Pupils perceiving purpose in learning.
2. Learners being involved in the solving of problems.
3. Meaningful learning experiences being inherent in the mathematics curriculum.
4. Provision being made to guide each learner in achieving optimal gains in ongoing units of study.

THE MATHEMATICS CURRICULUM

Which objectives should pupils achieve in the mathematics curriculum? Much emphasis is being placed upon the three R's (reading, writing, and arithmetic) in the school class setting. The third R (arithmetic) as a curriculum area certainly has utilitarian values in society. Each individual then makes use of arithmetic to buy needed goods and services in the economic world.

Problem Solving in Mathematics

Teachers, principals, and supervisors need to select relevant objectives for learners to attain.

Learners need to develop proficiency in computation. Meaning needs to be attached to each acquired addition, subtraction, multiplication, and division fact. Interest and purpose on the part of pupils is also important in achieving competency in the basics in the mathematics curriculum.

What is learned in computing needs to be applied in school and in society. Problem solving objectives then become significant. In society, problems are identified and relevant solutions sought. Thus, the school curriculum also needs to emphasize problem solving. For example, if new carpeting is to be placed in a classroom, pupils with teacher guidance may measure to determine the number of square yards or square meters needed. Comparisons may be made of the cost of carpeting from several businesses selling carpets. The total cost might then be computed in carpeting the classroom.

There are numerous additional practical experiences for pupils in the arithmetic curriculum. These include, among others, the following:

1. counting how many will be drinking milk, as well as how many will be eating in the school lunchroom at noon. The results may be graphed for each of the days of the week.

2. measuring ingredients in English or metric units to cook a representative

food dish of a specific country being studied in an ongoing unity of study.

3. helping to plan menus in the school lunch program. Adequate attention needs to be given to meeting proper dietary requirements, as well as amounts of food to be served per person.

Simulated experiences in problem solving also need to be provided for learners. Carefully chosen mathematics textbooks may provide relevant content in solving word problems. Other means of providing word problems for pupils include using selected duplicated worksheets and pages in workbooks. Pupils may also write word problems. Learners might then dramatize content contained in word problems. It is vital that pupils learn to transfer what has been learned in solving word problems to life-like situations in society.

Understanding Mathematics Concepts

Pupils need to attach meaning to what has been learned in ongoing units of study. Rote learning and memorization of subject matter is not adequate. Rather a variety of materials need to be utilized to guide each pupil to understand each concept developed in the mathematics curriculum. Among others, relevant concepts for pupils to attach meaning to include the following:

1. addition, subtraction, multiplication, and division.
2. commutative properties of addition and multiplication.
3. identity elements for addition and multiplication.
4. associative properties of addition and multiplication.
5. distributive property of multiplication over addition.
6. property of closure.
7. mathematical sentences.

To achieve understanding of a concept is ongoing. For example, a pupil does not attain understanding of the concept addition in one or several lessons. Rather, sequential, cummulative experiences are needed for pupils to achieve meaning involving vital concepts. Thus, each learner may acquire understanding of vital concepts involving continual progress during the public/private school years. Reputable textbooks, objects, slides, filmstrips, films, pictures, and study prints may be used as materials to provide learning activities in guiding pupils to understand significant mathematical concepts.

Decision-making and the Learner

Pupils with teacher guidance need to have ample opportunities to participate in the making of decisions. Life in society demands that individuals become proficient decision-makers. Teacher-pupil planning may be utilized to select objectives, learning activities, and appraisal procedures in the mathematics curriculum. Learners need adequate background information to be involved in planning ends and means of learning with teacher guidance.

A second method emphasizing input from pupils in determining content in the mathematics curriculum is to use learning centers. The teacher may obtain the materials and determine tasks for each center. However, each pupil may choose, sequentially, tasks to complete. There needs to be an adequate number of centers and tasks in order that pupils might omit experiences that do not meet personal needs, interests, and purposes.

A third means to involve pupils in ascertaining the mathematics curriculum is to utilize a contract system. The individual pupil with teacher guidance develops a contract. For example, the following contract might have been agreed upon with the due date for its completion:

1. Work each arithmetic problem on pages 50-53 with a minimum of 85% of items having correct responses.
2. Make a geoboard to show a triangle, rectangle, parallelogram, square, and rhombus. Correctly determine the perimeter and area of each geometric figure.

3. Complete an art project using geometrical figures in its content.

A fourth method involving pupils in decision-making in the mathematics curriculum is to utilize a mathematics laboratory means of instruction. With appropriate materials, pupils individually and in committees, may weigh, measure, as well as determine volume of selected items and objects. Learners do the choosing of which activity to pursue sequentially.

Evaluation of Pupil Achievement

There are numerous ways to appraise learners progress in the mathematics curriculum. A very significant means to evaluate achievement is to utilize teacher observation. What might a teacher observe pertaining to mathematics achievement of pupils?

1. correct or incorrect computation in addition, subtraction, multiplication, and division. Errors need to be diagnosed and remedied.
2. learner skills in being able to solve problems in mathematics in society.
3. understandings acquired of relevant concepts in the mathematical world.
4. attitudes possessed by pupils toward the mathematics curriculum.

Checklists may also be utilized to appraise pupil progress. The teacher needs to write vital objectives on a checklist for learners to attain. Learning activities are then provided to guide pupils to achieve the relevant ends. After instruction, the teacher may check if pupils individually have or have not achieved the chosen goals. The pupil's name and the date in using the checklist should be indicated. Notice the following checklist, as a model.

Name of Pupil______________________________Date________________

1. The pupil can accurately compute the area of a triangle.
2. The pupil can accurately compute the perimeter of a triangle.
3. The pupil voluntarily completes work in finding the area and perimeter of triangles.

After instruction, the teacher can check if a pupil has or has not achieved each stated objective. Additional assistance needs to be given each pupil who has not attained an objective.

Anecdotal records might be utilized to evaluate pupil performance. The teacher needs to follow the following standards in writing anecdotal records:

1. write representative behavior for each pupil being appraised. Do not write only negative or only positive behavior unless this is representative of a learner.
2. date the time the behavior is being recorded.
3. record each pupil's behavior frequently enough to notice patterns of responding.
4. use the results of anecdotal records to do a better job of teaching. A teacher may forget specific facets of each pupil's behavior unless it is recorded at random intervals.
5. do not use loaded words in recording representative behavior of each pupil. If a teacher writes "John is a trouble maker," biased terminology is being utilized. Uncertainty exists in terms of what is meant by the concept "trouble maker". Rather, the teacher needs to write exactly what John did.

Thus, the teacher needs to write narrative factual accounts of a pupil's behavior. The following anecdotal statements indicate written factual statements of behavior:

1. John completed eight of nine story problems with no errors in computation.
2. Arthur completed five of twenty addition facts. He could not be encouraged to do more.
3. Melvin and David visited with each other during time devoted to practice in mathematics. Each student completed ten of twenty-two subtraction computations.

4. Alice asked for an additional assignment in mathematics.

Pupils with teacher guidance may also write diary and log entries. Diary entries are written on a daily basis, whereas logs contain pupil experiences in mathematics for a longer period of time. The teacher may then notice what pupils experience in ongoing units of study. For example, pupils individually and in committees may record problems solved, concepts acquired, and basic facts mastered on a daily or weekly basis. The teacher might use diary and log entry results as a basis for determining new sequential objectives in mathematics. Or, results of pupil recorded information may be utilized to diagnose weakness in learner progress. Remediation activities might then be provided pupils to overcome identified deficiences.

In Conclusion

Pupils need to experience a relevant mathematics curriculum. Valid and reliable means need to be in evidence to ascertain pupil progress. Each learner needs to experience interest, purpose, and meaning in ongoing units of study. Definite provision needs to be made for individual differences among pupils.

MATHEMATICS AND THE LEARNER

Mathematics teachers and supervisors need to study trends in the curriculum and work in the direction of changing from what exists presently to what should exist in teaching-learning situations. The following are offered as trends for thoughtful consideration in the mathematics curriculum.

1. Certainly, pupils should use mathematics in everyday life. Learners buy selected goods and services at stores. Payment is made for what is purchased. Change is received from money left over after payment has been made. Teachers must find ways in which pupils might use in functional situations that which has been learned previously.
2. Learners need to understand number theory. Thus, as pupils progress through the school years, they will again and again at more complex levels understand, among other things, the commutative property of addition and multiplication, the associative property of addition and multiplication, the additive and multiplicity identity, the property of closure, as well as diverse number systems.
3. Geometry must receive ample attention in teaching-learning situations in the mathematics curriculum. Learners see geometry all around their environment. Buildings contain windows and doors shaped like squares and rectangles. Selected windows may have the configuration of a circle. Thus, geometry is near to us and is visible in everyday life. A quality sequential geometry curriculum needs to be available to all learners.

4. The mathematics curriculum must be individualized for pupils. Each learner is at a different achievement level as compared to others in a class. Thus, provision must be made for individual differences. There are several approaches, among others, which may be utilized in providing for each student in the classroom. The teacher may pretest learners to find present achievement levels in mathematics. Learners may then begin work at a specific place within a unit using a reputable series of mathematics textbook as a guide. Assistance is given to each pupil as needed.

The teacher may also develop a unique program of mathematics instruction for each pupil. Teacher -pupil planning may be utilized here whereby continuous progress in mathematics is in evidence. As another approach in providing for individual pupils, the teacher could have learners work using diverse algorisms in solving problems. Thus, for example, if learners are adding numbers such as the following: 64 + 27 + 18 = each child should determine which algorism is most meaningful for the personal self. A pupil could add the numbers from the ones column first and regroup the sum in terms of ones and tens. The tens could then be added to the tens column in the addition problem above. A second child could begin adding the tens from the tens column; this would be followed by adding the values from the ones column followed by regrouping so that any value of ten from the ones column can be added to other tens.

5. Pupils should have ample opportunities to develop learnings inductively. Thus, if pupils encounter a unique problem such as the following, they should be able to arrive at correct needed solutions:

Jimmy had fifty-five cents; he bought some kite string for 16 cents. How much money did he have left?

Various responses may be made by learners in determining how much money Jimmy had left. One child could suggest counting from 16 to 55 to state how many cents Jimmy had left. A second child may suggest counting backward from 55 to 16 to determine how many cents Jimmy had left. A third child may suggest a more sophisticated approach in arriving at the correct answer through regrouping and renaming such as in renaming 55 in terms of four tens and fifteen ones. Thus, six may be taken away from fifteen and one ten may be taken away from the four tens leaving three tens and nine ones or 39 as the answer to the original problem. Pupils then should have ample opportunities to develop learnings inductively or through discovery.

6. Mathematics laboratories may provide stimulating experiences for pupils. The laboratory should give pupils ample opportunities to experiment with diverse materieals using concrete, semi-concrete, and abstract materials. Thus, for example, pupils may measure the length, width, and height of selected objects and items of their own choosing. They might weigh items in the nearby environment pertaining to the use of grams along with its larger and smaller division of weight. Pupils also need to ascertain the volume of selected containers within the framework of functional

learning situations. In a mathematics laboratory then, pupils would be engaged in learning by doing rather than being passive individuals.

7. The concerns of pupils must be considered in an updated program of mathematics instruction. Thus, learners with teacher guidance may develop learning stations where pupils can be involved in making decisions pertaining to what is to be learned. At each learning station, pupils may select a task or tasks from among several on a task card. Learning stations such as the following may be in evidence:
    (a) reading center pertaining to materials on mathematics using library books, encyclopedias, and pamphlets.
    (b) filmstrip center where pupils individually or in small groups may view content pertaining to mathematics.
    (c) textbook(s) center where pupils may work word problems pertaining to selected content.
    (d) art corner with a variety of media whereby pupils may draw diverse geometric figures pertaining to people, buildings, and creative content in general.
    (e) writing center pertaining to pupils writing stories and poems relating to ongoing units of study in mathematics.

In Summary

Teachers, principals, and supervisors must study trends carefully pertaining to all curriculum areas in the public schools. Selected trends may be found by reading content from recent periodical articles and textbooks. Knowledge of curricular trends may also be obtained by attending professional meetings in education and speaking with other professionals involved in the teaching profession. Workshops and faculty meetings should emphasize relevant trends in teaching-learning situations.

CHANGE IN EDUCATION

There is considerable debate on the quality of education in the United States. The public schools have received a barrage of criticism from the lay public. These criticisms have included:

1. teachers are not teaching the basics.

2. frills and fads are offered too frequently in the curriculum.

3. schools are centers for vandalism, drug abuse, and crime.

4. graduates of high schools cannot function in the business world due to inadequacies in the curriculum.

5. students do not function well in society after completing the secondary school years.

Very little criticism is hurled at the following institutions in society:

1. the home. And yet forty percent of United States marriages end up in divorce. Child abuse appears to be in epidemic proportions in the home setting.

2. the business world. It is difficult for many workers to be able to secure employment. Unemployment figures are high in the United States. The private sector (business) has been very ineffective in providing jobs for involved persons. Companies providing employment, in numerous cases, seem to be on very shaky grounds. Selected enduring businesses lay off workers and at a later time rehire some of the laid off workers. Companies doing well in profit may relocate to a foreign nation due to lower wages being paid in the receiving nation.

3. the farm sector. The occupation of farming is indeed in deep trouble. Foreclosures of farms, high interest rates on farm loans,

grain embargos by the United States government, imports of foreign beef, among other factors, has made for highly complex and negative situations for farmers. The American farmer is very productive and produces a surplus of agricultural products, thus making for low prices for farm products. Productivity as a concept is applied to the business world to produce manufactured products at a lower price so that more consumers can purchase that which has been produced. And yet, the concept of productivity does not apply to farming since farmers definitely have been too productive and in the process ruined, in too many cases.

4. society itself. Crime and drug abuse certainly is rampid in American society. Prisons are definitely overcrowded and are seemingly places of terror. Little evidence, if any, exist in that penal institutions truly rehabilitate inmates.

5. religious institutions. Ministers misuse money coming from parishioners. A minister being involved in a love affair with a married woman was involved in the murder of her husband.

To be sure, there are excellent homes, businesses, societal situations, and religious leaders who do well and present a model to emulate. However, there is a need for institutions in society to appraise themselves and work toward improvement. Each institution needs to cast out the mote in their own eyes before trying to remove the motes from the eyes of others.

Proposed and Actual Changes in Education

Seemingly all states in the United States are rushing toward making for changes in the public schools. The writer believes that many of the modifications are ill conceived and hastily implemented. Florida, for example, in the 1984-1985 school year begins to rate teachers for merit pay (career ladder). Many educators have cautioned against the haste in that quality criteria have not been developed for appraisal purposes. It is easy to have the slogan "Lets cull out the weak teachers," but quite complex in its implementation. Too frequently slogans sound good and are accepted by naive individuals.

All professional groups have quality individuals as well as those who are not ethical. The following are examples:

1. medical doctors who sell drugs for strictly profit motives. In these cases, prescriptions are not written to help ill persons or those in pain. Newsreports of doctors selling drugs illegally is more and more commonly reported in the media.

2. outcomes in court trials can indeed be the ridiculous and the bizarre. Victory rather than justice and fairness too frequently are prized among lawyers and the legal profession.

3. cashiers and officers in banks being involved in embezzlement cases.

4. administrators and officers in corporations giving themselves huge salaries and bonuses. Too frequently, wage earners have their wages cut in half so that the corporation can "survive."

5. state legislators saying there is not money to fund education adequately, and yet raising their own salaries and pension plans a few months later.

Which changes in education then are being proposed? What are the pros and cons of these modifications?

The state of Mississippi is proposing to change how schools will be funded and the costs of education financed. Demonstrated, objective achievement from pupils will be used to finance schools in Mississippi. If a school system does not have students' achievement measure up to a specific level of accomplishment, funding can be curtailed or halted. Before funding is decreased, however, a school system will need to provide a plan and its implementation for remedying pupil deficiencies. This will be a probationary period. If at a later designated date, students do not reveal improvement in achievement, state aid can be curtailed. The involved school could even lose accreditation.

Advantages in funding schools based on student achievement could be the following:

1. teachers and administrators would emphasize that pupils achieve the basics.

2. pressure would be placed upon schools to measure up and have students achieve.

3. valid procedures would be utilized to appraise a schools effectiveness and that being student progress, rather than evaluating the number of library books and up-to-date textbooks a school has, how many teachers have a master's degree and higher for teaching, along with other extraneous factors.

Disadvantages in providing state aid based on demonstrated student achievement include:

1. disadvantaged students already achieve at a low rate due to lack of opportunities to learn. Should these pupils receive more state

aid than advantaged students so the former may progress more optimally? Deficiencies in learning may be minimized if materials, equipment, and adequate teacher assistance is available.

2. disadvantaged pupils generally are housed in poor quality school buildings. State aid needs to be forthcoming in order that decent structures are available for teaching and learning.

3. parental involvement in helping their low achieving offspring is definitely needed. It may cost money and additional teachers to teaching classes in parenting to students and parents.

President Reagan has emphasized the need for tuition vouchers. These vouchers would be available to parents in selecting the school and teacher for their child(ren). Thus, the involved parent would have a choice as to the kind of education desired for their child, whether it be public, private, or parocial education. The voucher would be in the form of state aid and redeemable for the child's education at the local school, or a more distant school, depending upon the objectives of education desired by the parent. Advantages given for the voucher system are:

1. parents are then involved in making decisions pertaining to their child's education.

2. parents should be knowledgable in terms of which goals in education would be best for their child(ren) to attain.

3. parents are responsible for their offspring and the quality of education of the latter should be no exception.

Disadvantages in emphasing a voucher means of education include the following:

1. parents may not know or care about which teacher truly could

provide for the best education of the involved child.

2. there may be a great imbalance in terms of numbers of pupils per teacher. If good teachers receive many students due to parental choice, boards of education would need to provide adequate assistance for the respected teacher. Teachers who are not chosen by parents frequently enough for their child's education may need to leave the teaching profession, even though quality raters would commend them for professional work done well.

3. the logistics may not work in moving pupils from the home school to those chosen by parents further away. If the school district pays for involved transportation costs, an added item of great expense could be involved. Should parents foot the bill for transportation costs, problems may arise due to single or both parents working in the world of work.

President Reagan has also advocated tuition tax credits for parents sending their child to a private or parochial school. Parents who send children to private or parochial schools would be able to deduct a certain amount of money from income taxes owed to the federal government. The state of Minnesota permits these deductions from their state income tax.

Reasons given for advocating tuition tax credits are:

1. parents pay double for having their children taught in nonpublic schools, such as paying the involved private or parochial school for education costs of the son or daughter as well as paying taxes to support public schools.

2. financing public schools would be very costly if private and parochial schools could not afford to stay open.

3. parents could do more of choosing a school, whether it be public, private, or parochial, if lower amounts of income tax were paid. The money saved could then be utilized to pay costs involved in private or parochial schools.

Disadvantages given for advocating tuition tax credits include the following:

1. church and state must be separated. It is strictly a parent's decision if nonpublic schools are to be patronized.

2. many private and parochial schools do not meet appropriate state standards. Thus, quality in education may be lacking.

3. public schools are free for all children, if parents desire to send their children here. There may be an excess in growth of unregulated private and parochial schools if tuition tax credits are enforced.

Prayer in the public schools, or meditation as it is sometimes called, continues to draw its supporters in Congress as well as from the White House. "God has been left out of our schools," according to advocates of prayer in the public schools.

Fundamentalist ministers advocate prayer in the public schools, as well as teaching of creationism. The concept of creationism is based on the first chapter from the Old Testament book of Genesis in the Bible. It appears that prayer and Bible reading are being advocated again in the public schools by selected individuals and groups in American society.

Reasons given for emphasizing prayer in the public schools include:

1. parents are not emphasizing religion in the home setting. Schools need to make up for these deficiencies.

2. currently the United States has the slogan, "In God we Trust." The House and Senate in the Capitol building open each session with prayer.

Freedom of religion is important for all in the United States, including public school classrooms.

3. school achievement of pupils has not been what it could be due to leaving God out of the public schools.

Opposing views on school prayer are the following:

1. the home and church need to emphasize religious instruction, rather than the public schools.

2. religious beliefs of each student may well differ from other learners in the classroom, thus hindering implementing group prayer efforts.

3. church and state must remain separate. Public schools are the reasponsibility of the state and not of diverse religious groups.

There are numerous additional issues in American education. Too often, advocates of change to "improve" the public schools have pretended as if issues do not exist. The changers believe that right and wrong practices are in evidence only.

Issues, other than those previously discussed in education, are the following:

1. more homework for students. It sounds as if teachers are responsible for not requiring enough homework. Too frequently, students do not complete homework assignments. Thus, it may be ridiculous to assign homework to students. Should homework in quality and amount truly be inadequate, parents need to be certain that their child's assignment is completed. The best work possible must be expected from each pupil.

2. teachers do not need more money. A beginning teacher may receive $12,000 a year salary. Salaries such as these reveal the lay public's

value systems. Notice the numbers of baseball, football, and basketball players that net a million or more dollars a year. Singers, actresses, and actors also are numerous in number who receive over a million dollars a year.

To pay for four years of college or university in receiving a baccalaureate degree and receive a beginning salary of $12,000 a year, the involved teacher certainly has provided a bargain to taxpayers. It is no wonder that selected teachers with school age pupils are eligible for food stamps and free school lunches.

3. test scores on the ACT and the SAT went downhill during the time the United States government provided federal aid to education in increased amounts. President Reagan has repeatedly made this statement. And yet, if proficiency goes down when federal money is provided for an institution, such as education, questions, such as the following arise:

1. Why increase defense spending the way it has been done? Does increased spending for the military lower defense and offensive capabilities?

2. Why do state senators and representatives, as well as members of the house of representatives and the senate increase their salaries? Applying the logic of test scores going down with increased federal funding, then performance by state and national legislators should go down also.

3. "Let the states and the locals make more decisions in the public school" has become a rather common slogan. In other words, deregulation has been emphasized as an important concept in American society. However, if deregulation is the goal to strive for in the United States, what will

happen to the environment (dioxin and other harmful chemicals which pollute and poison), to the drug traffic (certainly strong enforced rules need to be in evidence to minimize harmful drug imports and drug abuse among citizens in the United States), as well as unsafe products manufactured (automobiles whose rear wheels lock when brakes are applied)? The writer does not believe Americans want deregulation. They do and must want rules and regulations which truly protect the health, safety, and morals of its inhabitants. Federal regulations are a must and can be excellent for each to participate in attaining life, liberty, and the pursuit of happiness.

In Summmary

Educators, as well as the lay public, need to evaluate to determine if "change" in education and the public schools is positive, negative, or neutral. Too freguently, changes recommended sound good and thus become slogans. People in society should not believe that changes recommended necessarily will make for an improved curriculum.

However, it is costly to provide quality education for pupils in any society. Thus, teachers need to receive adequate remuneration for services provided. A pupil-teacher ratio needs to be in evidence in which teachers can truly assist students to achieve more optimally. Instructional materials must be adequate to provide for individual differences among learners. School buildings are costly to maitain and innovate when innovations are necessary. The lay public must realize that it costs much money to educate its citizens. Ignorance is very expensive and definitely unaffordable.

INSERVICE EDUCATION AND THE COMPUTER

With an increasing number of computers in the curriculum, staff development becomes important. Societal trends emphasize a continual emphasis being placed upon the utilization of computers in the business world, as well as in personal lives of individuals. The school curriculum must not be separated from society. Thus, the computer has a highly significant role to play in teaching-learning situations.

Workshops as Inservice Education

A theme for a workshop should be selected cooperatively by teachers with administrative guidance. The theme must reflect curricular needs of a school. One relevant need in the curriculum might well be computer utilization in teaching and learning.

A first level of participation in a workshop should involve all participants in a general session. The leader and involved individuals should then identify problem areas or facets of computer use that should be studied. Criteria to be followed in the general seesion include:

1. All should participate and no one dominate.
2. Each participant should stay on the topic being discussed and not stray to unrelated areas.
3. Participants should respect ideas being presented. Minimizing or ridiculing ideas presented definitely hinders achievement in communication.
4. Ideas need to be presented clearly and meaningfully among general session members.
5. Content expressed by individuals needs to circulate among members in a group, rather than between the leader and a respondent in sequence.

Which problem areas involving computer use might be identified as relevant to pursue? The following are provided as suggestions:

1. Which criteria need to be followed in selecting computers which harmonize with objectives of the school and class?
2. Which standards need utilization in choosing computer software?
3. How might computers be utilized in problem solving activities in the curriculum?
4. How might programmed learning be utilized to provide for individual differences?
5. Which guidelines need following to assist learners to attain optimally in using computerized drill experiences.
6. How might simulations and games involving computer usage assist students to develop decision-making skills?

After an adequate number of vital problems have been selected within the general session framework, participants may choose which committee(s) to work in. Each participant should select committee membership based on the following criteria:

1. Meeting personal needs to improve the curriculum in the class setting.
2. Promoting perceived purpose by the participant in solving vital problems in the classroom setting.
3. Stimulating interest in wanting to use computers to provide for individual differences.
4. Developing an attitude of wishing to utilize computers effectively in ongoing lessons and units of study.

An adequate number of reference sources need to be available to assist workshop participants to secure needed information in the solving of problems. These reference sources may include testbooks, periodical articles, pamphlets, films, slides, filmstrips, transparences, and illustrations. Proficient consultant and resource personnel also need to be available to participants in the solving of problematic situations.

In addition to general sessions and committee endeavors, individual participants also need opportunities to work on projects of their very own choosing. Thus, personal needs may be met in using computers in the classroom. A teacher, for example, may wish to develop his/her own programs for programmed instruction. Quality assistance needs to be available from an expert to guide teachers to develop their own programs of computerized instruction.

Achievements in committee work and individual projects may be shared with members of the total workshop within the framework of the general session. Whatever is achieved may, hopefully, be implemented in teaching-learning situations in the school/class setting.

In Conclusion

Inservice training of administrators and teachers is necessary to optimalize computer usage in the curriculum. Only then might quality objectives, learning activities, and appraisal procedures be selected to challenge student achievement in the curriculum.

INNOVATION, TECHNOLOGY, AND THE MATHEMATICS CURRICULUM

Change is a key concept in curriculum development. Mathematics instruction is no exception. Changes occur in society and thus related innovations need to follow in the school-class setting. Certainly, school and society should not be isolated from each other. Situations in the curriculum become practical and useful as learnings gained become applicable in the larger environment beyond the school setting. Which changes in society may provide meaningful trends in the mathematics curriculum?

Society and the Mathematics Curriculum

The mini-calculator as an invention has aided in bringing in many changes within the framework of the mathematics curriculum. Thus, primary and intermediate grade pupils, for example, may now obtain correct answers to complex addition, subtraction, multiplication, and division computations with the use of the mini-calculator. The answers, of course, may be determined rapidly and accurately by learners. Considerable enjoyment may well be experienced by learners in this situation. Compare this learning activity with the use of paper and pencil only, in dividing a six digit value (dividend) by a three digit divisor involving a remainder. Also the pupil may check the accuracy of the response by multiplying the divisor times the quotient and adding the remainder to the product. The latter procedure is truly time consuming and much frustration may be in evidence.

Many pupils, as well as selected school systems, own mini-calculators. They, of course, are then available for classroom use in ongoing units of study. With the introduction of a new piece of technology, such as the previously mentioned calculator, questions such as the following need thorough consideration and resolving:

1. what should be the role of the pocket electronic calculator in the mathematics curriculum?
2. how can pupils be guided to compute accurately in the areas of addition, subtraction, multiplication, and division using

pencil and paper? What implications are there for calculator usage in these basic operations on numbers?

3. which learning experiences assist pupils in attaching meaning and understanding to content in ongoing units of study?

4. how can pupil interest be implemented in achieving desired objectives in the mathematics curriculum?

5. what can the teacher do to implement teaching strategies emphasizing problem solving, critical thinking, as well as creative thinking? What role does the calculator play in emphasizing these processes?

Business and industry for some time have utilized management principles and procedures within the framework of diverse operations. Commercially prepared materials in the teaching of mathematics are available stressing these processes in diverse units of study. Teachers in various inservice education programs may also develop and utilize materials dealing with management systems of instruction in the mathematics curriculum. Perhaps, within the framework of a single or multiple series of mathematics textbooks, the teacher (or teachers) may utilize the following criteria to develop a management program of instruction.

1. write valid measurable objectives pertaining to content in the adopted series of textbooks.

2. develop a pretest to place learners in working on content appropriate to the achievement level of each learner.

3. a second pretest may also be given to place each individual learner in achievement within the confines of a unit.

4. the measurable objectives are then revised, if necessary, for each learner to attain sequentially. Each pupil is appraised in terms of having attained these ends in sequence.

5. a post-test may be given to each learner at the end of a unit. The teacher can then appraise progress in terms of each pupil.

Thus, in society, management methods and procedures have been utilized in business and industry. This is a societal trend and it has had its reflections and input in the school curriculum, including the mathematical arena.

A third societal trend pertains to individuals needing to be able to solve personal and social problems. This trend, no doubt, from the beginning of time of the human experience until now, has been important. A futuristic curriculum will also proclaim the importance of stressing problem solving in the mathematics arena. Thus, within the framework of a stimulating environment, learners identify problems and work in the direction of obtaining related solutions. The mathematics laboratory concept of teaching as one method or approach, could well emphasize the development of problem solving skills within the learner. In this kind or type of learning environment, pupils would actually measure, weigh, determine volumes of selected containers, as well as find areas and perimeters pertaining to identified life-like problems. Materials needed to perform these tasks must be available at one or more learning stations. Problems are identified by pupils with teacher guidance. Assistance is given to pupils as the need arises in arriving at solutions directly related to these problem areas.

Since problems then need identifying and related solutions developed in society, the mathematics curriculum also must emphasize problem solving objectives for pupils to achieve. Other societal trends which may well have relevant implications for the mathematics curriculum include the following :

1. human beings are economic individuals. They basically earn a living and buy needed goods and services. Pupils need to be guided in thinking about relevant careers and how money earned may be spent wisely. Creative thinking would be an integral part of this learning activity.

2. the annual federal and state budgets allot moneys to separate categories for spending. Based on accurate background information, pupils may hypothesize on suggested modifications of categories as well as total budget figures.

3. pollution in its diverse forms has been of major concern to individuals in society. Learners may be guided in thinking about the role of mathematics in offering solutions to these problem areas.

4. inflation is perceived as being a major problem to consumers. Pupils with teacher guidance at the appropriate stage of development may provide suggestions in terms of needed curbs to inflationary trends.

5. the state sales tax, federal and state income taxes, social security payments, and other forms of taxing individuals also may well provide content for a functional mathematics curriculum for pupils. Thus, school and society become integral and not separate entities.

In Closing

Teachers and supervisors need to study trends and situations in society. These societal trends may well suggest objectives, learning activities, and evaluation procedures in the mathematics curriculum.

KNOWLEDGE OF RESULTS

Ralph Tyler[1] asks four questions pertaining to developing the curriculum. These are:

1. Which objectives should pupils achieve?
2. Which learning activities need selection to help pupils achieve the desired goals?
3. How should the curriculum be organized?
4. Which techniques should be utilized to evaluate pupil achievement?

This paper will focus upon evaluation procedures to utilize in appraising pupil achievement. However, all four facets of curriculum development are interrelated. Thus, within the framework of learning activities (reading materials, films, slides, transparencies, filmstrips, pictures, and other audio-visual materials used in instruction), the curriculum may emphasize in organization:

1. the separate subjects curriculum
2. the correlated curriculum
3. the fused curriculum
4. the integrated curriculum.

How successful have the learning activities been to guide pupils to achieve objectives? The teacher will need to evaluate if learners have been successful in achieving stated objectives.

Evaluation Procedures

There are numerous techniques that teachers may utilize to appraise learner achievement. Ragan and Shepherd[2] list the following techniques:

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[1] Ralph Tyler, Basic Principles of Curriculum and Instruction, The University Of Chicago Press, 1950.

[2] William Ragan and Gene Shepherd. Modern Elementary Curriculum, 5th edition. New York: Holt, Rinehart and Winston, 1977, pp. 453-458.

1. Standardized Achievement Tests
2. Teacher-made Tests
3. Personality tests and rating scales
4. Sociograms
5. Teacher observation

Not all techniques of evaluation measure or appraise the same or similar facets of learner growth. Standardized achievement tests tend to measure subject matter learnings acquired by pupils. There are a few exceptions to that statement, e.g. The Iowa Tests of Basic Skills, as the name indicates, attempts to measure achieved skills rather than facts, concepts, and generalizations attained by pupils.

Teacher written tests tend to lean in the direction of assessing learner achievement in measuring subject matter achieved by pupils.

Personality tests attempt to measure achievement in attitudes and values acquired by pupils. Rating scales, also, tend to appraise attitudinal changes. Thus, a pupil can be evaluated on a 5 point scale, if he/she is:

(a) working cooperatively with others.

(b) respecting contributions of other learners.

(c) completing assignments on time.

Sociogram results provide knowledge of results to the teacher if a pupil is growing in social development. The following questions may be asked of pupils to which learners respond individually in writing:

1. If you were asked to play a game with others, who would be your first, second, and third choices?
2. If you were to do a reading research project together with three other pupils, who would be your first, second, and third choices?

Teacher observation may be utilized to appraise:

1. pupil behavior in informal and formal situations in the school setting.

2. the quality of art, construction, dramatization, and research projects completed.

3. the strengths and weaknesses of daily assignments completed by learners.

Jarolimek[3] lists the following techniques to utilize in evaluating pupil achievement:

1. group discussions
2. observation
3. checklists
4. conferences
5. anecdotal records
6. work samples
7. experience summaries
8. diaries and logs
9. sociometric devices
10. teacher-made tests
11. standardized tests

Jarolimek lists selected evaluation techniques which are the same as those given by Ragan and Shepherd. These are observation, sociometric devices, teacher-made tests, and standardized tests. Those which differ include group discussions, check lists, conferences, anecdotal records, work samples, experience summaries, diaries, and logs. Ragan and Shepherd, however, do list teacher observation as an evaluation technique which may be inherent in each of the immediately above listed techniques. Thus, when pupils participate in group discussion, the teacher might observe which pupils are participating effectively as compared to those lacking knowledge and skills in group procedures. Also, in the use of a checklist, the teacher may observe which pupils are and are not attaining desired listed behaviors. A checkmark may then be made for the learner who has achieved one or more of the listed goals. The teacher, moreover, can observe progress in responses given by pupils individually and collectively within a conference. The following evaluation techniques also require the utilization of teacher observation:

1. anecdotal statements written by the teacher. To write and date representative behaviors of any pupil behavior, the teacher needs to observe carefully specific incidences in the behavior.

2. work samples completed by pupils. Through teacher observation, the completed art activities and written reports,

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[3] John Jarolimek. *Social Studies in Elementary Education*, 5th ed. New York: The Macmillan Company, 1977, pp. 349-361.

among other products, may be evaluated. Thus, for example in completed pupil written work, the teacher may evaluate the quality of subject matter, spelling, capitalization, usage, and handwriting.

3. experience summaries, diaries, and logs. Experience summaries cover subject matter learned and is written by pupils individually and collectively at selected intervals of time. Thus, the teacher may observe content acquired by pupils in the experience summaries reflecting ongoing units of study. Diary entries are written on a daily basis, and log entries, generally, for a longer period of time, covering a week in terms of duration in time. Again the teacher through observation may notice in the written work what pupils have/have not attained.

Teacher observation may be considered a subjective, rather than an objective evaluation technique. For example, in evaluating written products of pupils, evaluators would not agree as to what is inherent in a quality piece of writing. Selected teachers might place major emphasis upon quality ideas when appraising written work. Perhaps, little emphasis is placed upon the mechanics of writing by the evaluators. Others, in appraising pupil written work might place high value upon the mechanics of writing, such as correct spelling of words and proper usage (choice of words) in writing using standard English.

Behaviorism And The Evaluation Process

James Popham, a behaviorist, from the University of California believes strongly in the utilization of measurably stated objectives in teaching pupils. The following are examples of these precisely stated ends:

1. The pupil will add correctly nineteen out of twenty addition problems.
2. The pupil will answer correctly nine out of ten multiple choice items.
3. The pupil will write a one hundred word paper on causes of inflation.

The teacher then needs to choose learning activities to help each pupil achieve that which is stated in the objective, e.g. being able to add correctly, responding to multiple choice items accurately, and writing a paper on causes of inflation. Each objective has a mimimal level of acceptance or achievement for a learner to be successful in attainment. Learning activities selected by the teacher should guide pupils to achieve only what is inherent in each objective.

After instruction, the teacher may measure each pupil's progress to ascertain if sequential objectives have been attained. For unsuccessful pupils, a new teaching sequence needs to be developed. Thus, each pupil may receive feedback on how well he/she is achieving.

James Popham[4] makes the following recommendations in providing knowledge of results to pupils:

1. "a simple right or wrong" response given by the teacher to pupils' answer.
2. "a simple statement of the correct response" given by the teacher so pupils may check their own answers.
3. "right or wrong" response given by the teacher "plus elaboration" deemed necessary to pupils' answer.
4. "the correct response plus elaboration" by the teacher in reply to pupils answers.

Popham[5] also believes that pupils should have knowledge of results as soon as possible pertaining to completed assignments, as well as voluntary endeavors. Knowledge of results should come at least within one hour after a learner has completed achieving a measurable objective.

Popham[6] further presents a classroom model of reinforcement emphasizing knowledge of results:

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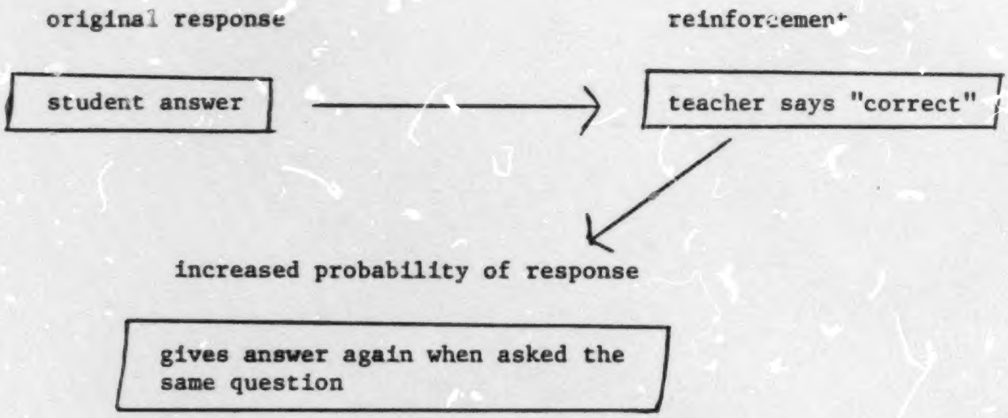
[4] James Popham, Knowledge of Results, (filmstrip and cassette), Vimcet Associates, 1969.

[5] Ibid.

[6] Ibid.


original response
reinforcement
student answer
teacher says "correct"
increased probability of response
gives answer again when asked the same question

There are selected questions which might be asked pertaining to Popham's suggestions and model for teaching:

1. Can relevant goals for learner attainment be stated in measurable terms as to attitudes, creative endeavors, and decision-making?
2. The classroom model for reinforcement was developed from a study of animal behaviors in a carefully controlled laboratory setting. Might the results be or not be transferable to human endeavors in the school and classroom setting?
3. May knowledge of results to pupils be given by the teacher in terms of "that's right" or "that's wrong"? Creative behavior and attitudes of learners cannot be appraised in terms of being good or bad.
4. Should pupils individually be involved in appraising their own personal progress in learning?

Humanism And The Curriculum

Humanists advocate pupils being involved in decision-making procedures in ongoing units of study. A learning center's philosophy may then be in evidence. Pupils might then sequentially choose which tasks to pursue sequentially, as well as which to omit. The teacher may provide the materials and tasks for each center. Pupil-teacher planning might also be utilized to determine the contents of each learning center. Additional methods also emphasizing humanism, as a psychology of learning, include:

1. a contract system. Through pupil-teacher planning, the tasks for the former to complete may be written up in contract form.

- The due date and attached signatures of the pupil and the teacher may accompany the contract.

2. pupil-teacher planning for individual and committee projects for the former to complete. Diverse audio-visual materials might be utilized to initiate a unit. The teacher attempts to stimulate pupils to identify questions during the initiating activities. Each question is listed on the chalkboard or on a transparency. Pupils may then volunteer to find information on a selected question or questions.

3. emphasizing learnings on getting to know the personal self and others better. A variety of learning experiences may be in the offing to learn about feelings, values, and beliefs that each person has.

4. involving the learner in evaluating personal achievement. The teacher should not be the only being involved to evaluate pupil progress. Thus, the involved pupil assists in appraising personal achievement to obtain knowledge of results.

Morris and Pai[7] write the following pertaining to humanism as a psychology of learning:

> In an attempt to counteract the dehumanizing impact of the technological approach to education, humanistic educators point out the crucial need and importance of focusing our attention on individuals--their interests, needs, and aspirations. Hence, they regard education as a process of self-actualization in which "one makes oneself." Indeed, people are not merely engineering products, for they do have something to say about what they become.

In Conclusion

Who should provide knowledge of results to the involved learner? Should this be the teacher's prerogative or should pupils be guided to appraise their own progress? How specific may knowledge of results be provided to pupils? Do knowledge of results in spelling achievement differ from knowledge of results emphasizing creative endeavors? Teachers and supervisors need to analyze and attempt to synthesize teaching and learning procedures involving the behaviorism/humanism dichotomy. Both are recommendable psychologies to emphasize in developing the curriculum. Each pupil needs to perceive interest, purpose, and meaning in ongoing units of study.

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[7] Van Cleve Morris and Young Pai. Philosophy and the American School. Second Edition. Boston: Houghton Mifflin Company, 1976, page 406.

EVALUATING ACHIEVEMENT IN MATHEMATICS

Pupil achievement in mathematics must be appraised in terms of objectives for ongoing units of study. Thus, in teaching-learning situations, first of all, the teacher must select objectives carefully which learners are to attain. Secondly, learning activities must be chosen which will guide pupils in achieveing desired ends. Finally, the teacher must appraise pupil progress in terms of stated objectives.

Using Teacher Observation

The teacher through careful observation may evaluate pupil achievement. The teacher may observe traits such as the following in the area of achievement in elementary school mathematics:

1. pupils completing assigned work on time.
2. interest learners reveal in mathematics.
3. continuous progress made by pupils.
4. specific errors pupils make in mathematics.
5. pupils understanding new learnings.
6. additional practice learners may need in understanding a new process in mathematics.
7. how to group pupils for instruction.
8. the learning styles of individual pupils.

Thus, the classroom teacher can evaluate many facets of pupil achievement in mathematics using the technique of teacher observation.

Using Checklists

The classroom teacher may wish to utilize checklists to appraise pupil performance in mathematics. Many of the teacher's observations may be forgotten unless they are recorded. With the use of a checklist the observations of the teacher can be recorded for each learner. The teacher in developing a checklists must, first of all, write out the behaviors that learners are to be appraised in. The following is an example of a checklist containing relevant behaviors in a specific ongoing unit of study:

Name of pupil____Date_______________

1. The pupil can satisfactorily add two one-digit addends.
2. The learner can reproduce with markers values of two single-digit addends.
3. The pupil appears to enjoy working on addition problems containing two one-digit addends.
4. The learner is becoming increasingly independent in his/her school work.

The teacher should write the pupil's name and the date of appraisal on each checklists. Thus, the teacher may notice specific weaknesses learners exhibit in an ongoing unit of study and attempt to remedy these deficiencies. Comparisons may be made of each learner's progress from past times to the present when checklists are used.

Using Teacher Made Tests

Teacher-made tests may be utilized to appraise pupil performance in the mathematics curriculum. Teacher-made tests should be valid. Thus, the test will attempt to measure what pupils have had opportunities to learn during class sessions devoted to mathematics instruction. The tests should also be reliable. For example, if pupils took the same teacher-made test over again, results should be comparable from the first time to the second time. This would be true if circumstances were equivalent in taking the teacher-made test the second time. Thus, consistency of results is important when thinking of the concept of reliability in testing.

The following, among others, are examples of items which may be included in a teacher-made test providing pupils have opportunities to engage in previous practice of these learnings:

1. simple computation of two one-digit addends-- 9 + 5 = _____, 8 + 7 = _____, 4 + 8 = _____, 5 + 9 = _____, 7 + 8 = _____, and 8 + 4 = _____.

2. word problems involving computations studied previously-- John had nine marbles; he bought five more. How many marbles does he now have?

3. multiple choice items-- Which of the following is NOT true:

a. 17 + 18 = 18 + 17 c. 35 - 17 = 18

b. 35 - 17 = 35 - 18 d. 35 - 18 = 17

Teacher written test items must follow criteria recommended by specialists in the area of testing and measuring of pupil progress. Thus, test items written by the teacher must be

1. valid and reliable.
2. clearly written.
3. on the understanding level of pupils.
4. properly ordered for pupils in ascending order of difficulty.

Essay items may also be written by the teacher to appraise pupil achievement in elementary school mathematics. Essay tests should only be utilized as an appraisal technique if

1. pupils have an adequately developed writing vocabulary.
2. learners possess an adequate reading vocabulary to comprehend the content of the test items.
3. pupils have had ample learning experiences to respond effectively to essay items in the test.
4. the items are properly delimited and yet do not require factual recall of information. For example, an essay item requiring the following

response would be too broad--Discuss mathematics. Certainly, the pupil is entitled to know what facet of mathematics learning he/she is to respond to. The following would be too narrow in scope to be called an essay item--Write a definition for the term "sum" as used in addition. Essay items should reflect pupils' skill in problem solving: How does knowledge of the distributive property of multiplication over addition help in solving related problems in mathematics?

5. pupils can be evaluated in terms of selecting and organizing major ideas when developing desired responses to the test question. The teacher may also wish to evaluate pupils in terms of the mechanics of writing such as
   (a) spelling and handwriting.
   (b) capitalization and puctuation.
   (c) grammar and usage.

It is recommended by the writer that the mechanics of writing be assessed separately from ideas expressed by learners when responding to different essay items. Pupils, of course, can work on remedying a few deficiencies in the mechanics of writing at a given time. Objectives that pupils are to achieve should be attainable and not overwhelming.

Using Standardized Tests

Standardized tests which are valid and reliable may be utilized periodically to appraise pupil achievement in mathematics. Pupils' results from taking of standardized tests need to be evaluated in terms of the following questions:

1. Did the learner respond correctly to important concepts or word problems? If not, what remedial measures would be important to remedy identified deficiencies?

2. Were errors made in the area of computation? If so, specifically which kinds of errors were made? What provision need to be made to correct these deficiencies on the part of learners?

3. Did it appear that pupils were interested and properly motivated in taking the test?

Pupils' reults from standardized tests will generally be given in terms of the following results:

1. grade equivalent score. Thus, for example, a pupil in the beginning of the fourth grade may obtain a raw score equivalent to the 5.7 grade level. A different fourth grade pupil may be achieving on the 2.9 grade level as indicated by results from the standardized test. Regardless of the present grade level of the pupil, the raw score obtained by learners from taking the standardized test is converted to a grade quivalent as indicated in the manual for administering the test.

2. percentile ranks. A pupil, for example, in the fourth grade may have obtained a raw score of 45 (number of items responded to correctly) on a standardized mathematics test which is equivalent to the 45th percentile according to the manual for administering the test. Being on the 45th percentile would indicate that 45 percent of the pupils on the fourth grade level would score lower on the test than the previously named pupil. Fifty-five percent of the fourth graders taking the same standardized mathematics test would score higher. The 45 percent and the 55 percent figures mentioned previously would pertain to pupils whose test results were utilized as norms in developing the standardized test.

In utilizing standardized achievement tests to appraise pupil achievement in mathematics, the following facets of pupil growth are generally evaluated:

1. computation in arithmetic. Thus, pupils respond to test items involving addition, subtraction, multiplication, and division.
2. concepts in mathematics. The standardized test in mathematics may contain items pertaining to (a) the meaning of fractions, (b) inverse operations such as subtraction undoing addition, (c) place value in decimals, (d) the Roman system of numeration, (e) number systems,

(f) estimating, (g) understanding place value and (h) geometric terms.

3. word problems. Learners exhibit proficiency in analyzing which facts are needed in a problematic situation to obtain a needed solution.

> A family on a vacation traveled 325 miles the first day, 460 miles the second day, and 411 miles the third day. How many miles were traveled in the three days?

Thus, in the above problem, learners must glean necessary content, such as adding 325 + 460 + 411, to be able to arrive at the correct answer. Word problems can contain content, of course, on different levels of complexity.

Standardized achievement tests provide different results pertaining to pupil progress as compared to teacher-made tests.

1. The teacher may notice how learners in the class setting compare with pupils in the standardization groups as indicated by results from the standardized achievement test. The manual of the standardized achievement test should indicate the pupil population utilized in developing norms for the test. Thus, the teacher may compare the achievement of his/her own pupils with that of the group used to develop norms for the standardized achievement test.

2. The teacher may also notice the spread of scores, from high to low, of pupils in the class setting as indicated by results from the standardized achievement test. The spread of scores in the class setting may provide needed data to the teacher in providing adequately for each pupil in the mathematics curriculum. Each learner should, of course, be guided to achieve to his/her optimum in elementary school mathematics.

3. Specific kinds of errors that pupils reveal in elementary school mathematics, as indicated by standardized achievement test results, can provide relevant diagnostic data with the end result being to remediate deficiencies.

4. Pupil progress in mathematics may be noted as reliable and valid standardized mathematics achievement tests are given learners at selected intervals.

Using Anecdotal Records

The teacher must record an ample number of observations made about each pupil's achievement in mathematics. It is recommended that observations be recorded of each pupil's progress frequently enough to notice a pattern of behavior. The teacher of mathematics may notice the following from recorded information (anecdotal records) about a particular child's progress:

1. The learner does not respond accurately to basic addition and subtraction facts, such as 9 + 8 =___, 8 + 9=_____, 17 - 8 =_____, and 17 - 9 =_____.

2. The pupil is slow in getting started on assigned work in mathematics.

3. The child is easily distracted from working in mathematics by happenings in the environment.

4. The pupil seeming does not enjoy mathematics as much as other curriculum areas in the elementary school.

It is easy for a teacher to forget how each child is progressing in mathematics unless observations are recorded systematically. The recorded items or anecdotal records should be utilized to improve instruction for each child. Viewing the above numbered items, the teacher may remedy deficiencies by (a) giving the child meaningful practice in addition and subtraction facts which need mastering, (b) guiding the pupil to develop appropriate work study habits, (c) attempting to develop a learning environment conducive for pupils making continuous progress and (d) providing interesting, understandable, and purposeful experiences for pupils in elementary school mathematics.

The results of anecdotal records should be utilized to guide pupils to achieve continuous progress in elementary school mathematics. Anecdotal records should not be used (a) to defame pupils, (b) for gossip purposes, or (c) as a threat to involved learners.

Using Work Samples

A manila folder to contain work sample of each pupil in mathematics is highly recommendable. Thus, within a manila folder for a particular child, the teacher may save representative samples of a child's work pertaining to elementary school mathematics. The completed papers should be dated before being placed in the folder. Thus, comparisons can be made by the teacher as well as the pupil of the latter's present as compared to earlier achievement in mathematics. Too frequently, the teacher has felt that a specific child is not achieving well in mathematics until comparisons are made between and among earlier and later work in mathematics.

Pupils should evaluate their own achievement in mathematics using work samples. Thus, a child can appraise his own progress by noticing if he/she is doing better than formerly when comparing results from earlier times to the present. Thus, in adapting the mathematics curriculum to the present achievement level of each learner, the pupil individually can take pride in noticing achievement from earlier times to the present when appraising completed products in this curriculum area.

Using Parent-Teacher Conferences

The teacher may obtain valuable feedback from the parent or parents as to understandings, skills, and attitudes pupils have developed in elementary school mathematics. Criteria to follow in conducting parent-teacher conferences include (a) respecting the thinking of parents, (b) wishing to obtain information about a child's progress in mathematics, (c) working together with parents to develop a good mathematics curriculum for each learner, (d) keeping information obtained from parents confidential, (e) agreeing with parents as to future courses of action pertaining to a relevant mathematics curriculum for each pupil and (f) revealing positive attitudes toward parent-teacher conferences.

Work samples of pupil's completed products in mathematics may be shown and discussed with parents in a conference. Thus, parents have opportunities to notice

(a) present achievement level of the child.

(b) kinds of errors made by the individual pupil.

(c) rate of progress accomplished by an individual child.

(d) neatness of work revealed by the learner.

Conducting a parent-teacher conference is not an end result in and of itself. The results of a parent-teacher conference should aid in developing a meaningful, purposeful, and interesting mathematics curriculum for each child. Individual differences must be provided for in a modern program of elementary school mathematics!

Using the Tape Recorder

A tape recorder can be wisely used in the class setting to appraise pupil achievement in mathematics. The following, among others, are major ways in which tape recorders may be utilized to appraise pupil performance:

1. Have pupils listen to a playback pertaining to their involvement in a discussion of a new process in elementary school mathematics.

2. Date and file tape recordings of pupils' discussions in learning a new process in elementary school mathematics. Periodically, pupils with teacher guidance may notice growth in mathematics achievement when comparing earlier with later recordings.

3. Have pupils discuss ways to improve the mathematics curriculum. Pupils may listen to a replay of the recording to evaluate criteria given. The teacher may obtain valuable feedback from the recording in terms of content for improving the mathematics curriculum.

Diagnosing Pupil's Responses in Mathematics

The teacher must be a good diagnoser of pupil achievement in elementary school mathematics. Thus, the teacher identifies specific kinds of errors that pupils individually are exhibiting in elementary school mathematics. If a pupil, for example, continually responds incorrectly to the sum of 6 + 5, the following reasons may be inherent:

1. The pupil may need to attach meaning to concepts pertaining to a set of six and a set of five. Crayons, beads, chalk, books, bean and corn seeds, as well as buttons may be utilized to have pupils see and duplicate a set of six markers and a set of five markers. The two sets may then be joined together to form a new set of eleven members.

2. The learner may need practice in committing the addition fact "6 + 5=11" to memory. A variety of experiences should be utilized in providing practice for pupils pertaining to selected learnings in elementary school mathematics. Dull, boring, and routine learning activities are to be avoided.

3. Pupils may even need to work on learnings in elementary school mathematics where they are at a less mature level than what is called for in the problems "6 + 5 = 11". Thus, for example, the pupil may need guidance in rational counting. The teacher could help the individual pupil to utilize the concept of one-to-one correspondence when counting six members in a set. The proper counting number as it is said would then correspond with the object being counted in the set consisting of six members, i.e., count "one" as the first object is being touched, "two" for the second object, "three" for the third object, and so forth, until all six members have been counted.

The teacher of elementary school mathematics must be a good diagnoser of specific difficulties that pupils face in ongoing units of study. Only then can remediation efforts be applied to guide pupils to overcome identified problems in the mathematics curriculum.

Using Conferences to Appraise Pupil Progress

The teacher may conduct conferences with elementary school pupils in order to evaluate achievement in mathematics. Criteria to follow in conducting these conferences include the following:

1. Respect the contributions of each pupil.
2. Permit free expression of ideas so that the teacher may obtain feedback

from each pupil's thinking about ways to improve the curriculum.

3. Try to find out specific help that pupils need in order to achieve optimal development in mathematics.

4. Agree upon a plan of action to help each pupil achieve relevant understandings, skills, and attitudinal objectives in elementary school mathematics.

After the conferences has been completed with a pupil, the teacher should (a) briefly record important ideas gained from the learner, and (b) determine how ideas gained from the conference can provide input for an improved mathematics curriculum for the involved learner.

The teacher may conduct a conference lasting, approximately, five to ten minutes for a pupil each day. It does not take long before all pupils in the class setting have been involved in a conference. For example, if there are 25 pupils in a class, the first conference could be completed for each pupil in five weeks' period of time. The conference approach can be one method, among others, utilized to appraise learner achievement in elementary school mathematics.

In Summary

There are numerous techniques available to appraise pupil achievement in elementary school mathematics. The teacher must utilize a variety of approaches to assess pupil growth in the mathematics curriculum. Each evaluation technique has its strengths as well as weaknesses. Thus, a specific evaluation technique may be utilized as a check on other approaches to appraisal. Pupil achievement must be assessed in terms of stated relevant understandings, skills, and attitudinal objectives. It is not adequate to appraise pupil growth in terms of understandings objectives only. Pupils must also be assessed in terms of skills objectives. The understandings acquired by learners must be utilized; thus, skills objectives need to be stressed adequately in ongoing units of study in elementary school mathematics. Adequate emphasis also needs to be placed upon pupils achieving attitudinal goals. Desireable attitudes on the part of learners aid in achieving understandings and skills objectives. A defensible program of evaluation would then stress that pupil achievement be adequately appraised in terms of understandings, skills, and attitudinal objectives.

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